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A REVIEW OF A CASE AGAINST BLAISE PASCAL AND HIS HEIRS

David A. Schum*

THE PROBABLE AND THE PROVABLE. By *L. Jonathan Cohen*. New York and London: Oxford University Press. 1977. Pp. 380. \$22.00.

I. INTRODUCTORY COMMENTS

A recent book by Professor L. Jonathan Cohen of Oxford University, *The Probable and the Provable*, provides new insights into the concept of probability in the intellectual task of inductive inference, a species of human reasoning about which there is perpetual controversy. The terms *inference* and *probability* recur throughout jurisprudence. The fact finder in criminal or civil proceedings, for example, must often make inferences about the truth of facts when the evidence is incomplete, inconclusive, or unreliable. This task, as Wigmore, for one, concluded,¹ is usually inductive and can be viewed as using observable evidence to draw conclusions about general propositions or facts. The fact finder, in other words, must weigh evidence or balance probabilities. As Maguire *et al.* remark at the very beginning of their casebook on evidence:

Evidence is produced at a trial so that an impartial trier can decide how an event occurred. Time is irreversible, events unique, and any reconstruction of the past [is] at best an approximation. As a result of this lack of certainty about what happened, it is inescapable that the trier's conclusions be based on probabilities.²

Cohen offers a view of inductive inference which is rooted in the works of Sir Francis Bacon and John Stuart Mill. Cohen's view of inductive inference leads to a new conception of probability, one, he believes, especially congenial to juridical inference. In fact, he applies his new probability system, which he terms *inductive probability* (P_I), to the Anglo-American legal system.

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1. J. WIGMORE, *THE SCIENCE OF JUDICIAL PROOF* 20 (3d ed. 1937).

2. J. MAGUIRE, J. WEINSTEIN, J. CHADBOURN & J. MANSFIELD, *CASES AND MATERIALS ON EVIDENCE* 1 (6th ed. 1973).

Cohen contrasts his system with what he calls *mathematical probabilities* (P_M). Mathematical probability was initiated by the seventeenth-century French mathematician Blaise Pascal in his studies of games of chance. Since Pascal's time many individuals have revised and extended mathematical probability systems which are highly useful in the study of a wide variety of random processes. Although Cohen recognizes the richness of mathematical probability systems and their applicability to many scientific areas, he notes their inadequacies for, and questions their applicability to, juridical inference.

Cohen frequently refers to P_I measures as Baconian probabilities to acknowledge that they are part of the legacy of Bacon's seminal work on inferential methods of science. Similarly, P_M measures are frequently called Pascalian probabilities. As the title of this review suggests, I will address Cohen's views concerning the inadequacy of Pascalian P_M measures and the suitability of Baconian P_I measures in applications within jurisprudence and related areas.

The Probable and the Provable assumes a fairly high level of knowledge of formal logic and probability theory. However, sections of this book which relate to inference in jurisprudence can, I believe, be mastered by those who have some aversion to or lack of interest in mathematics. In this review I offer two sets of comments about Cohen's book, one for those averse to mathematical symbols and one for those without such an aversion. I believe Cohen's work is important and will influence future studies of juridical inference. Consequently, this book deserves a hearing from the largest possible audience.

II. NONTECHNICAL REVIEW

It is common to think of probabilities in connection with a wide variety of phenomena in law, medicine, science, and business. Mathematics has developed a rich system of probabilities useful in these areas. Although there has been some disagreement about what probability statements mean and about how numbers called probabilities are determined, there has been essential agreement among mathematicians about the basic properties of probabilities. Probabilities are numbers between zero and one. If there are two events that cannot both happen at the same time, the probability of one or the other happening

is the sum of their separate probabilities. If two events can both occur, the probability of their joint occurrence is determined by multiplying the probabilities of the separate events. In my introductory comments I gave the label P_M to probabilities which have these three properties. These simple algebraic properties have formed the basis of a rich and extensive probability system which now helps solve many remarkably difficult and subtle problems.

Rich food sometimes causes indigestion, and Cohen argues that the richness of the P_M system causes intellectual "indigestion" when mathematical probabilities are applied to inferences at trial. The symptoms of such indigestion appear in the form of paradoxes: Some of these paradoxes arise because of the *multiplication* rule within the P_M system which determines the probability of the joint occurrence of two or more events, while others arise because of a special case of the addition rule which determines the probability that one of two events has happened, if they both cannot have happened. This special case is called the *negation* rule and concerns mutually exclusive events like "rain tomorrow" and "no rain tomorrow" (which are exhaustive, since it will either rain or not rain tomorrow). The event "no rain tomorrow" is the logical complement or the negation of the event "rain tomorrow." If I tell you that the probability of rain tomorrow is 0.8, I must, within the P_M system, tell you that no rain tomorrow has probability 0.2. The more likely "rain" is, the less likely "no rain" is under the rules of the P_M system, since probabilities for complementary events must total one.

Cohen argues that the multiplication rule for P_M is inconsistent with the forensic standards of proof in civil cases and with the many interpretations of the legal requirements for "inference upon inference." In such inferences, you must, for example, prove B from C and then prove A from B. Concerning the "preponderance of evidence" or "balance of probability" rule in civil cases, Cohen argues that the multiplication rule within P_M allows a plaintiff to lose his overall case on the balance of probability even if each element or component of the case was won on the balance of probability. That is, the joint or overall case consists of element 1 *and* element 2 *and* element 3 *and* so on. Under the P_M system, the probability of the overall case (given the evidence before the court) is obtained by multiplying the probabilities of each of the elements (given evi-

dence relevant to each element). Even assuming that the probability of each element is much greater than 0.5 (to satisfy the preponderance of evidence rule), the *product* of numbers less than one sooner or later drops below 0.5. Thus, the overall contention could have probability less than 0.5 even though every element in the contention had probability greater than 0.5.

Moreover, when you multiply 0.8 times 0.4 you get the same result as when you multiply 0.4 times 0.8; the order in which multiplication is performed makes no difference. This causes difficulties in relating P_M to the frequent requirement that in "inference upon inference" in civil cases where B is proved from C and then A is proved from B, A can be proved from B "on balance of probabilities" but B must be proved from C "beyond a reasonable doubt." Thus, the A, B "link" in the chain of inference can be weak provided that the B, C "link" is very strong. Because order of multiplication is irrelevant, the P_M system will give the same result if the A, B link were strong and the B, C link were weak.

Other difficulties arise, says Cohen, because the negation rule requires that probabilities for complementary events must equal one within the P_M system. Cohen argues that this requirement makes a trial appear to be the division among the litigants of a fixed amount of case weight; the more plaintiff receives the less defendant receives. He believes this is unfortunate since the strength of the plaintiff's arguments may not necessarily detract from the strength of the defendant's arguments. In short, he views a trial as a test of case strength and believes that the P_M system encourages the view that one litigant's gain is the other's loss. Finally, Cohen tells us of other assorted difficulties of applying P_M to "prior presumption of innocence" and "beyond a reasonable doubt" prescriptions in criminal cases and to more specific problems concerning corroborative and convergent evidence.

A major portion of *The Probable and the Provable* is devoted to developing a new system of probabilities Cohen calls "inductive probabilities" and which I symbolize P_I . Cohen's essential claim is that his new system more adequately reflects the way ordinary persons actually reason inductively. Inductive probabilities, although they rest upon developments no less carefully reasoned than mathematical probabilities, have simpler, more primitive, properties. In fact, they do not behave at all like P_M measures: they cannot be added, subtracted, multiplied, or di-

vided. Although evidence can change the probability of an event, we cannot say, for example, that event A is twice as likely as event B, given relevant evidence. In fact, we can only make what are called *ordinal* or *ordering* relations among inductive probabilities. I can say that, on the evidence, A is more probable than B, but I can neither say how much more probable nor how many times more probable.

The P_I system lacks the richness of P_M . What do we receive in return? Cohen argues that various prescriptions and standards in the Anglo-American legal system are not meant to satisfy mathematicians or philosophers; they are meant to guide the reasoning of ordinary citizens when they serve as factfinders. He further argues that the basic underpinnings of his P_I system better reflect the ordinary inductive reasoning individuals use in their day-to-day affairs. One of his claims is that the results of psychological experiments involving probability estimations are better explained in terms of the P_I system rather than the P_M system. In addition, he shows how the various paradoxes mentioned above are not encountered in the P_I system. Unlike the P_M system, for example, the P_I system allows a plaintiff to win a civil suit if and only if each element in the suit is established on the balance of probability. Moreover, in civil suits involving "inference upon inference," the P_I system is not insensitive to the ordering of strength among links in the inferential chain and in any litigation the P_I system captures the trial as a test of strength rather than as a division of case merit.

If you read further, you will see that I do not agree with all of Cohen's conclusions. The elegant simplicity of P_I , however, illuminates a variety of evidentiary and other inferential issues about which there is unending controversy. I feel safe in predicting that *The Probable and the Provable* will be an influential work in future studies of juridical and other inferences.

III. TECHNICAL REVIEW

For those of you remaining with me I will now be a bit more specific in telling you about what I believe are the important aspects of Cohen's work.

A. *Mathematical Probabilities: A Legacy from Pascal*

Let us begin by taking a brief look at certain developments and interpretations of P_M in order to identify those features of P_M that, according to Cohen, cause difficulties when it is applied in various contexts, including juridical inference. In games of chance the determination of probabilities is an exercise in counting. In such games there is usually some "visible symmetry" of the underlying process which reinforces the assumption of equally likely outcomes; fair dice, well-balanced roulette wheels, and well-shuffled cards are examples of random processes which have this property of visible symmetry. Sometimes the assumption of equally likely outcomes is called the indifference principle; if a die is fair you should be indifferent about the likeliness of its six possible outcomes when the die is thrown. In such cases the probability of event E is determined by counting the number of outcomes favorable to the occurrence of E and dividing this number by the total number of outcomes possible. Probabilities so determined are sometimes called *classical probabilities*; rules for their determination date from the time of Pascal. Such determination is not possible when there is not a finite number of possible outcomes or when there is no reason to assume equally likely outcomes.

The so-called *relative frequency* interpretation of probability extends P_M to make it applicable to random processes in which the outcomes are not necessarily equally probable and not necessarily finite in number. In such cases the probability of an event is estimated empirically. Assuming a random process which is repetitive or replicable, the probability of an event E is estimated by determining the frequency of outcomes favoring this event in a randomly chosen sample of n outcomes. The probability of E is *estimated* by the ratio of the number of outcomes favoring E to the total number n of outcomes in the sample. The limit of this relative frequency in a long run of randomly chosen samples is taken to represent the probability of E .

A relative-frequency interpretation of P_M does rather well as long as we wish to apply probability to processes which are repetitive in nature. Many random processes in various areas of science are repetitive in nature and so a relative frequency interpretation of probability is congenial to these areas. However, we often wish to express our uncertainty about the occurrence of an event which either has happened or will happen *exactly*

once. In many legal, medical, business, and military affairs it is necessary to evaluate the likeliness of events in nonrepetitive or nonfrequentistic processes; the events of concern are unique or one of a kind. The probability that your horse wins tomorrow's race is a one-of-a-kind event since this race and its outcome will happen exactly once. The event that defendant X was seen running from the house where the crime was committed is a unique event. Now, you will have no trouble in finding someone who will express probabilities about various outcomes of tomorrow's race. Neither will you have difficulty finding a treatise on evidence law which talks about probabilities in connection with evidence that involves patently unique events and nonrepetitive outcomes. Under another interpretation of P_M you are free, subject to certain rules for coherency, to assign numbers called *subjective* or *personal* probabilities which indicate the strength of your belief that some repetitive or nonrepetitive event in question has occurred or will occur. Adherents of this interpretation argue that your subjective probability of an event can be supported by whatever information you have.

Other interpretations of P_M are possible, including a *causal* or *propensity* interpretation in which P_M grades the measurable physical connection between objects or processes. Perhaps the most complete summary of various interpretations of P_M is to be found in a recent treatise by Fine.³ Cohen has performed a valuable service by showing how various probability statements are, in fact, evaluations of inferential soundness and he offers a useful categorization scheme for P_M based upon various criteria for establishing inferential proof.

Following are seven properties of P_M ; the first three are basic or axiomatic. For each property we shall consider both *unconditional* or *monadic* probabilities of the form $P_M(E)$, where E is some event of interest, and *conditional* or *dyadic* probabilities of the form $P_M(E|F)$, which is read "probability of event E given that or on the premise that event F occurred." By definition $P_M(E|F) = P_M(E \cap F)/P_M(F)$, provided that $P_M(F) \neq 0$. The conjunction or intersection symbol $\cap$ is read "and." The terms monadic and dyadic are Cohen's terms; for reasons discussed in his second chapter he objects to the term "conditional probability."

3. T. FINE, THEORIES OF PROBABILITY (1973).

- (i) For any event E , $P_M(E) \geq 0$; for any events E and F where $P_M(F) \neq 0$, $P_M(E|F) \geq 0$.
- (ii) Let S be the "sure" event; one that is certain to occur. Then, $P_M(S) = 1$, and for any E where $P_M(E) \neq 0$, $P_M(S|E) = 1$.
- (iii) Suppose events E and F are mutually exclusive, then $P_M(E \cup F) = P_M(E) + P_M(F)$. Assuming $P_M(G) \neq 0$, $P_M(E \cup F|G) = P_M(E|G) + P_M(F|G)$. This is the so-called *additivity* property. [The union or disjunction symbol $\cup$ is read "or."]
- (iv) Let E^c be the complement of event E (i.e., $E =$ "not E "). Since $E \cup E^c = S$ and E and E^c are mutually exclusive, $P_M(E) + P_M(E^c) = 1$. Assuming $P_M(F) \neq 0$, $P_M(E|F) + P_M(E^c|F) = 1$. This is the *negation* rule.
- (v) For any two events E and F , $P_M(E \cap F) = P_M(E) \cdot P_M(F|E)$ where $P_M(E) \neq 0$. If $P_M(G) \neq 0$, $P_M(E \cap F|G) = P_M(E|G) \cdot P_M(F|E \cap G)$. This is the *conjunction* or *product* rule.
- (vi) Suppose event E fails to condition or change opinion about the likeliness of F ; i.e. $P_M(F|E) = P_M(F)$. In this case, events E and F are said to be *independent* and $P_M(E \cap F) = P_M(E) \cdot P_M(F)$. Suppose $P_M(F|E \cap G) = P_M(F|G)$; this asserts that events E and F are independent *conditional upon* the occurrence of G . In this case the product rule for conditionally independent events is $P_M(E \cap F|G) = P_M(E|G) \cdot P_M(F|G)$, provided $P_M(G) \neq 0$.
- (vii) Let H_1 and H_2 be mutually exclusive events where $P_M(H_1) \neq 0$, $P_M(H_2) \neq 0$. For any event E where $P_M(E) \neq 0$, $P_M(E|H_2) = 0$, and $P_M(H_2|E) \neq 0$,

$$\frac{P_M(H_1|E)}{P_M(H_2|E)} = \frac{P_M(H_1)}{P_M(H_2)} \times \frac{P_M(E|H_1)}{P_M(E|H_2)}$$
 This is the "odds-likelihood ratio" form of *Bayes' s rule*, a derived property of conditional or dyadic probabilities. The left-hand term represents the posterior odds of H_1 to H_2 given E . The first term on the right is called the prior odds of H_1 to H_2 . The second term on the right is called the likelihood ratio of event E .

Properties (iv) through (vii) represent a small sample from

the very rich collection or syntax of properties derived from the three basic axioms. As we shall see, Cohen argues that the negation and conjunction properties which are basic to P_M give rise to disturbing paradoxes when P_M is applied in jurisprudence. He further argues that these paradoxes are not evident under his inductive interpretation of probabilities.

It is easily shown that classical and relative frequency interpretations of P_M are consistent with the properties listed above. In fact, the basic axioms of P_M were motivated by the methods used to determine classical probabilities. Are subjective or personal probabilities consistent with the axioms and other properties listed above? Of course it depends on whose subjective probabilities we are discussing. Consider the event E that the Yankees will repeat as World Champions in the 1979 season. You have your view of the likeliness of this event and I have mine. Subjectivists argue that, though we may reasonably disagree about $P_M(E)$, our probabilities ought to be coherent within the three basic axioms mentioned above or else some unpleasant things can happen. If I say, for example, that $P_M(E) = 0.2$ and that $P_M(E^c) = 0.6$ (in violation of the negation property) I can be exploited as a "money pump." This means that certain wagers can be constructed in which I am guaranteed to lose no matter what happens. Thus, subjectivists usually confine discussion to individuals who are "coherent" with the basic axiom system of P_M . Whether or not persons, left to their own devices, are thus coherent is an interesting empirical issue, one generating a substantial amount of research in experimental psychology during the past two decades.

There are many methods for eliciting subjective probabilities from individuals. Some involve indirect methods in which subjective probabilities are inferred from a person's choices among wagers. Other methods are direct and involve overt elicitation of probabilities, odds, and likelihood ratios. As you might expect, coherency with P_M axioms is sometimes observed and sometimes not. Cohen argues that the actual responses of subjects in some of the research in this area are coherent within P_I but not within P_M . In short, he argues that P_I is much better than P_M in *describing* the way in which people actually assess the likeliness of events in their everyday lives. He also tells us that it is no accident that legal prescriptions regarding probability seem to favor a P_I interpretation since these prescriptions have in

mind the ordinary citizen who must apply his own reasoning processes and accumulated experience in evaluating evidence.

I have one final point about P_M and the properties listed above. When one adds probabilities such as in Property (iii) or takes ratios of probabilities such as in Property (vii), there are implicit assumptions about the scale properties of P_M . In all P_M systems that I am aware of, $P_M(E)$ is a number in the closed interval $[0,1]$. Adding probabilities assumes equal units of probability and dividing probabilities assumes equal units plus a true or nonarbitrary zero point. Though P_M always has a lower limit of zero in every system I am aware of, including P_I , there is plenty of room for argument about what zero probability means, as the author discusses. It is plausible to think that P_M is an equal interval scale for classical and relative frequency versions of P_M , but what about subjective versions? Is the change in your subjective likeliness of from 0.67 to 0.68 equivalent to the change of from 0.43 to 0.44? This is a serious issue for me since most of my rebuttal arguments against some of Cohen's conclusions involve subjective judgments of *ratios*. I will return to this issue in a later section of this paper.

B. *Inductive Probabilities: A Legacy from Bacon and Mill*

At the basis of Cohen's system of inductive probability is his belief that probability is a measure of inferential soundness or provability. Because there are different kinds of proof rules, there are different kinds of probability. Thus, if we say that the probability of E on the premise F indicates the degree of inferential soundness of a rule for inferring E from F we shall have to accept different interpretations of this probability since there are various ways in which such proof can be established. I have already mentioned the author's categorization of P_M measures based upon various proof criteria. Thus, the author emphasizes from the very outset of his work that his system of P_I measures is not intended to replace other conceptions of probability. What he does argue is that there is a particular proof criterion which, if considered, leads one to a new system for grading the inferential soundness of inferring E from F and, thus, grading the probability of E given knowledge that F occurred. This new system of inductive probabilities does not conform to the calculus

of P_M . Though there are a few points of contact between P_I and P_M , P_I is not derivable from P_M nor is it in any sense a special case of P_M .

A brief comment about notation is necessary before I proceed. In discussing P_M I have used notation that is common in many treatises in which P_M measures occur. The monadic or unconditional probability of E was indicated by $P_M(E)$; the conditional or dyadic probability of E on premise F was indicated by $P_M(E|F)$. When I discuss Cohen's P_I measures I will use his notation scheme to help prevent the reader from being confused about which system is being discussed. In the P_I system, the monadic probability of E is represented by $P_I(E)$ and the dyadic probability of E on the premise F is represented by $P_I(E, F)$.

A neglected criterion of proof, according to Cohen, concerns a property he calls "completeness." A deductive system is said to be *complete* if statement E is provable from the axioms of the system if and only if E^c is not provable. Applied to P_M we can consider the provability of E from F as a limiting case of P_M where $P_M(E|F) = 1$. The P_M system is consistent with this property of completeness since, by the negation rule, $P_M(E|F) = 1$ if and only if $P_M(E^c|F) = 0$; that is, E is provable from F if and only if E^c is not provable from F .

In an *incomplete* deductive system there is at least one well-formed statement E such that neither E nor E^c is provable. In such a system we allow for cases in which both the probability of E on premise F and the probability of E^c on premise F are zero. This obviously rules out P_M as a measure of provability in an incomplete system since $P_M(E|F) = 0$ and $P_M(E^c|F) = 0$ is incoherent with the P_M axioms. One question is: Why should we have to consider an incomplete system? Suppose, as the author argues, we construe the "weight" of relevant evidence to mean the *amount* of relevant evidence as Keynes⁴ and others have done. Keynes proposed that argument E has more weight than argument E^c if E is based upon a greater amount of relevant evidence; *i.e.*, E has greater probability than E^c if the evidence on balance favors E . The gradation of provability or the probability of E , given evidence F , depends just on the amount of evidence. Within the P_I system, if the amount of

4. J. KEYNES, A TREATISE ON PROBABILITY 71-78 (1921).

evidence F on balance favors E , $P_I(E, F) > 0$. If F provides an increment of inferability to E it provides none to E^c and so $P_I(E^c, F) = 0$. From the point of view that weight of evidence means amount of evidence, F cannot support E and E^c at the same time. Here we have a crucial distinction between P_M and P_I . In the P_I system if $P_I(E, F) > 0$ then $P_I(E^c, F) = 0$; the negation principle for P_I is not complementary as it is in P_M . In fact, in a P_M system, as $P_M(E|F)$ increases, $P_M(E^c|F)$ decreases since $P_M(E|F) + P_M(E^c|F) = 1$.

We must also notice that the converse of the P_I statement "if $P_I(E, F) > 0$ then $P_I(E^c, F) = 0$ " is *not* true. Evidence F may be completely indecisive or irrelevant on E and on E^c and so both $P_M(E, F)$ and $P_M(E^c, F)$ may be zero. Thus if we say that E^c is not provable from F we cannot necessarily say that E is provable from F since it may be the case that neither E nor E^c is provable from F . The author further argues that in many everyday matters we must reason inductively from incomplete evidence and what we need is a measure of how extensive is the completeness or coverage of relevant issues by the evidence we have.

As we shall see, the P_I measure has scale properties that differ from those of P_M . In fact P_I has only rank-ordering or *ordinal* properties. In this system we can only say that $P_I(E_1, F_1)$ is greater than, less than, or equal to $P_I(E_2, F_2)$. We cannot say how much greater or how many times greater is $P_I(E_1, F_1)$ than $P_I(E_2, F_2)$. This rules out any kind of additivity and it rules out the formation of ratios of probability measures. As we observed, the additivity of P_M requires an equal unit, and, ratios of P_M require equal units and a true or nonarbitrary zero point. Thus, as far as scale properties are concerned, P_I is a more primitive measurement system than is P_M . As we have seen, P_I has neither of the two main properties of P_M ; P_I is neither additive nor complementational. Professor Cohen does not view with alarm the fact that P_I measures are more primitive than P_M measures. In fact he believes that the ordinal characteristics of P_I better account for human inferential behavior and for the manner in which various legal prescriptions are written.

The next questions to be asked are (a) How does one determine the gradation of provability or inductive support offered by evidence F to proposition E ? and (b) How does this

gradation of support influence the probability of E on the premise F? The author devotes six chapters (chapters 12-17) to these matters. The foundation of P_I rests upon certain developments within inductive logic concerning the concept of inductive support. Let us first see how P_I forms part of the intellectual legacy from Bacon and Mill. It was Sir Francis Bacon who, in his treatise *Novum Organum* (1620), first attempted to formulate and justify inductive procedures for natural science.⁵ He argued that a proposition or generalization could not be validated simply by enumerating evidential instances favoring the proposition but it could be invalidated by a single unfavorable instance. He thus proposed a method of induction by elimination. Some hypothesized cause must be co-present, co-absent, or covariant to some degree with corresponding effects. Inductive inference, according to Bacon, proceeds by using presence, absence, or degrees of covariation of effects in order to eliminate various hypotheses about causes until perhaps only one hypothesized cause is left; the one that survives this process of elimination can be accepted as the valid cause. This procedure, of course, reminds one of a canon for reasoning much favored by another sage, Sherlock Holmes, who is reported to have said:

It is one of the elementary principles of practical reasoning that when the impossible has been eliminated the residuum, however improbable, must contain the truth.⁶

John Stuart Mill proposed a collection of specific methods for induction in his treatise *System of Logic*. Most present-day students of experimental design in various areas of behavioral, biological, and physical sciences study extensions of Mill's methods without being aware of it. Mill is frequently not given appropriate credit for systematizing the design of empirical research. A variety of procedures exist for introducing various experimental "controls" so that one can isolate valid causes by removing the confounding effects of other possible alternative causes. Cohen tells us that the process of grading inductive support that one proposition can give another has a close affinity to three of Mill's methods for induction. The *method of agreement* establishes the co-presence of a cause and effect; the *method of difference* establishes the co-absence of a cause and effect; and the *method of concomitant*

5. W. KNEALE, *PROBABILITY AND INDUCTION* 48 (1949).

6. A. DOYLE, *THE ANNOTATED SHERLOCK HOLMES* 17 (W. Baring-Gould ed. 1967).

variation establishes the covariation of a cause and effect.

Suppose a situation in which we entertain a particular hypothesis H_j which explains a characteristic of some phenomenon of interest; how do we obtain inductive support for H_j ? There may, of course, be other plausible hypotheses or explanations. Imagine now a series of tests which can discriminate among alternative hypotheses. Each test involves some relevant variable which can be manipulated independently of all others. The complexity of the test sequence increases as we proceed because at each stage a new relevant variable is added to those already present. As the test sequence proceeds, some hypotheses are falsified by test results and are eliminated from consideration. Suppose H_j survives the process of elimination. The degree or grade of support given by the test sequence to H_j depends upon the complexity level of the test that H_j attains. At some point we run out of relevant variables to manipulate or we run out of time or money and so we stop testing; the surviving hypothesis or hypotheses win the day in this process of eliminative induction. Mill's methods for induction provide the essential logic for the design of the test sequence.

Formally, Cohen identifies a *support function*, $s(H, E)$, which is read "The support for H , given test result E ." Suppose there are n test levels 1, 2, 3, ..., i , ..., n which represent increasingly complex tests. If H resists falsification or elimination up to test level i we can say that the grade of inductive support for H , given test result E , is $s(H, E) = i/n$. Test result E gives the i th grade of support where n is the highest grade possible. The support function value $s(H, E) = i/n$ says that H has support up to level i and no higher; *i.e.*, H was falsified at level $i + 1$. Suppose the test sequence is replicable; if you object to the test result the person performing the test sequence says "do it yourself." Suppose you do and achieve the same result. Replicability, in short, provides a measure of confidence in the test result. A test sequence replicated enough times becomes, as the author says, a "solid evidential fact." Thus, if $s(H, E) \geq i/n$ on the basis of replicable or "solid" evidence we are entitled to conclude that $s(H) \geq i/n$; *i.e.*, we can talk about the support for H without having to qualify it with a particular test result. Suppose $s(H_j, E) = 0$; H_j is falsified by the simplest test. Hypothesis H_k , however, passes test i but is falsified by test $i + 1$; $s(H_k, E)$ remains at level i/n and does not drop to zero because

H_k obviously has more support than H_j which had none at all.

It is always possible, and the author cites examples of when it has happened, that a theory or proposition H may be true but fail to explain certain effects. Anomalies do occur and any theory of induction must be able to handle them. It would seem foolish to suppose that no theory could remain acceptable when confronted with counterevidence. Perhaps, with some slight modification, theory H can be rescued and resist being falsified at some level; one can buy support for H by revising it.

So far we have an inductive support function $s(H, E)$ which maps ordered pairs of propositions H, E into $n + 1$ fractions from 0 to n/n , where $s(H, E) = 0$ means that H is falsified by the simplest possible test and $s(H, E) = n/n$ means that H is not falsified or eliminated throughout the entire test sequence and has the highest level of support. The value of n may not be specifiable in any practical application. Technically, this presents no problem since it is apparent that the support function assigns values with only ordinal properties since there is no apparent equal unit of "test level difficulty" specifiable. Thus $s(H, E)$ only *ranks* evidential support; the numbers thus obtained are not additive nor can they be used to form ratios.

The support function $s(H, E)$ has a number of properties, two of which are of special interest in view of the author's concerns about P_M measures.

- (i) *Negation.* For any H and E , where E concerns a physically possible event or conjunction of events, if $s(H, E) > 0/n$ then $s(H^c, E) = 0$.

This property may seem inconsistent with what was said earlier about the support given H when there is counterevidence at level $i + 1$; i.e., $s(H, E)$ stays at level i/n and does not drop to zero. The negation rule asserts that E cannot favor H and H^c at the same time; if it favors H then $s(H^c, E) = 0$. Observe that H and H^c are contradictory statements; if a test sequence which E reports appears to favor both sides of a contradiction the test sequence itself must be inconsistent and requires adjustment.

- (ii) *Conjunction.* Suppose H_1 and H_2 are two propositions and E is some report of a physically possible test sequence. If $s(H_1, E) \geq s(H_2, E)$, then $s(H_1 \cap H_2, E) = s(H_2, E)$.

The conjunction principle asserts that the support given by E to the joint statement $H_1 \cap H_2$ is the same as that given to the less-well-supported of the two propositions or to their common support level if they are equal.

The remaining question concerns how inductive support and inductive probability are related. The author tells us that $P_I(H, E)$ stands in relation to $s(H, E)$ as deducibility stands in relation to logical truth. The inductive probability that a *particular* thing is H , on the premise that it is E , is equivalent to the inductive support for the generalization that *anything* if it is H is also E . The corresponding symbolic assertion is that $P_I(H_{aj}, E_{aj}) = i/n$ if and only if $s(H_{aj} \rightarrow E_{aj}) = i/n$. The symbols like H_{aj} are read "particular a_j is an H "; the arrow means "logically implies." The more *support* you have for a generalization involving H and E the higher is the probability of H given a particular instance of E . The author cautions us that the evidential processes leading to an assessment of $P_I(H, E)$ are not necessarily the same as those leading to $s(H, E)$. The establishment of $s(H, E)$ rests upon suitably varied experimental or observational conditions which are replicable. The establishment of $P_I(H, E)$, however, rests upon satisfying ourselves that E favors or supports H in the particular circumstances which E and H describe.

Inductive probabilities have a number of basic and derived properties which are summarized in a syntax in chapter 17. Following are properties of special interest.

For Monadic Probabilities:

- (i) $P_I(H) \geq 0$.
- (ii) If $P_I(H) > 0$, then $P_I(H^c) = 0$. (Negation)
- (iii) If $P_I(H_1) \geq P_I(H_2)$, then
 $P_I(H_1 \cap H_2) = P_I(H_2)$. (Conjunction)
- (iv) $P_I(H_1 \cup H_2) \geq P_I(H_1)$. (Disjunction)
- (v) $P_I(H_1) \geq P_I(H_2)$ or $P_I(H_1) \leq P_I(H_2)$.
- (vi) If $P_I(H_1) \geq P_I(H_2)$, and $P_I(H_2) \geq P_I(H_3)$,
then $P_I(H_1) \geq P_I(H_3)$.

For Dyadic Probabilities:

- (i) $P_I(H, E) = P_I(H^c, E^c)$. (Contraposition)
- (ii) If $P_I(H, E) > 0$ and $P_I(E^c) = 0$, then
 $P_I(H^c, E) = 0$. (Negation)
- (iii) If $P_I(H_1, E) \geq P_I(H_2, E)$, then $P_I(H_1 \cap H_2, E)$
 $= P_I(H_2, E)$. (Conjunction)

- (iv) $P_I(H_1 \cup H_2, E) \geq P_I(H_1, E)$. (Disjunction)
- (v) $P_I(H_1, E_1) \geq P_I(H_2, E_2)$ or $P_I(H_2, E_2) \geq P_I(H_1, E_1)$.
- (vi) If $P_I(H_1, E_1) \geq P_I(H_2, E_2)$ and $P_I(H_2, E_2) \geq P_I(H_3, E_3)$, then $P_I(H_1, E_1) \geq P_I(H_3, E_3)$.

Like P_M , P_I has far too many derived properties to be easily summarized. Following are some facts about P_I which are important in relating P_I to inferential issues in jurisprudence and other related areas.

(1) According to Cohen the monadic or prior inductive probability $P_I(H)$ tells us something about the strength of nature's potential for bringing H about and it has a completely different interpretation from $P_M(H)$. The essential point is that $P_I(H) = 0$ means simply that there are no prior reasons for believing H and *not* that there are prior reasons for believing H^c . For P_M , however, $P_M(H) = 0$ if and only if $P_M(H^c) = 1$ which indicates, according to the author, that there are prior reasons for believing H^c . The monadic negation principle asserts that if $P_I(H) > 0$, then $P_I(H^c) = 0$. For P_I , if the weight (amount) of your prior reasons supports H it cannot, at the same time, support H^c .

(2) Properties (iii) and (iv) for monadic and dyadic inductive probabilities concern conjunction (intersection) and disjunction (inclusive union). These properties for P_I may be compared with their counterparts in P_M by considering the following inequalities. For monadic probabilities, where H_1 and H_2 are any two events: $P_M(H_1 \cap H_2) \leq P_M(H_1) \leq P_M(H_1 \cup H_2) \leq P_M(H_1) + P_M(H_2)$. For dyadic probabilities, where H_1 , H_2 , and E are any three events with $P(E) \neq 0$: $P_M(H_1 \cap H_2 | E) \leq P_M(H_1 | E) \leq P_M(H_1 \cup H_2 | E) \leq P_M(H_1 | E) + P_M(H_2 | E)$.

(3) The reader familiar with measurement theory will have observed that properties (v) and (vi) for both monadic and dyadic P_I concern weak-ordering. These properties are similar to the connectedness and transitivity axioms of subjective value theory.⁷ Subjective value is an ordinal measure of the worth or value of an outcome in a decision task.

(4) Property (i) for dyadic P_I is interesting. It says that

7. R. KEENEY & H. RAIFFA, DECISIONS WITH MULTIPLE OBJECTIVES: PREFERENCES AND VALUE TRADEOFFS (1976).

dyadic P_I is invariant under contraposition of its arguments (H and E). This is not a characteristic of P_M since $P_M(H|E)$ need not equal $P_M(H^c|E^c)$.

(5) One of the most interesting and surprising properties of P_I is the manner in which the inductive probability of H changes in response to favorable and unfavorable evidence. Consider H and H^c , supposing a priori that the weight of our prior reasons for believing H is no heavier than the weight of those favoring H^c . Then, $P_I(H) = P_I(H^c) = 0$. Remember that, within the P_I system, "weight" means "amount." Now suppose we have evidence E_1 which supports H ; then $P_I(H, E_1)$ equals, say, $1/n$; by the negation principle $P_I(H^c, E_1) = 0$. Next, E_2 is presented which is unfavorable to H . This makes $P_I(H^c, E_1 \cap E_2) = 2/n$ while, surprisingly, $P_I(H, E_1 \cap E_2) = 0$; again this is what we require by the negation rule. Evidence E_3 arrives which "counteracts" or explains away E_2 and H is resuscitated since $P_I(H, E_1 \cap E_2 \cap E_3) = 3/n$ while $P_I(H^c, E_1 \cap E_2 \cap E_3) = 0$ as required. Thus there is a marked asymmetry in the effect upon H that favorable and unfavorable evidence produce. Each item of uncounteracted favorable evidence increases the inductive probability of H one grade higher. However, an uncounteracted piece of evidence immediately reduces the inductive probability of H to zero. If this seems violently counterintuitive to you, the reason perhaps is that you are retaining P_M in your head while reading about P_I . Zero probability in the P_I system means something entirely different than it does in the P_M system. In the P_M system $P_M(H) = 0$ or $P_M(H|E) = 0$ make H "legally dead" or beyond resuscitation even by evidence overwhelmingly supportive of H . Recall that, within P_M , $P_M(H) = 0$ if and only if $P_M(H^c) = 1$, and $P_M(H|E) = 0$ if and only if $P_M(H^c|E) = 1$. A zero probability for H in the P_I system simply indicates that an inference to H cannot be made with any degree of support; it does not mean that an inference to H^c can be made with complete support.

C. The Case Against Pascal and His Heirs

If a formal or mathematical system is used to study how some task *ought to be* performed it is said to be used normatively. Normative statements prescribe formally ideal behavior

or behavior that is consistent with the basic axioms of the system. A formal system can also be used in efforts to study how some task is *actually* performed; in such cases the system is used descriptively. The author's essential claim is that significant difficulties arise when existing standards for forensic proof as well as certain other evidentiary issues in the Anglo-American legal system are construed in terms of the P_M system.⁸ Thus, his major arguments against P_M concern how well P_M describes these actual standards and prescriptions in the law as it is written. He goes one step further by claiming that P_M is similarly deficient in describing human behavior in certain probability estimation tasks studied by experimental psychologists. He then proceeds to show why he believes that the P_I system better describes probabilistic aspects of our legal system and better accounts for actual human behavior in some of the experimental studies of probability estimation. Summarized below are six specific difficulties or paradoxes the author says we encounter when existing legal prescriptions are construed in terms of P_M . He believes that these difficulties, though not completely insurmountable for P_M , are handled more easily and naturally within P_I .

1. *Conjunction Rule Difficulties in Civil Suits*

Consider a civil suit in which plaintiff's contention H consists of a number of points or elements $H_1, H_2, \dots, H_n$; i.e., $H = (H_1 \cap H_2 \cap \dots \cap H_n)$. Defendant's contention is $H^c = (H_1 \cap H_2 \cap \dots \cap H_n)^c = (H_1^c \cup H_2^c \cup \dots \cup H_n^c)$. In a civil suit plaintiff must prove each element of his case on the balance of or on the preponderance of probabilities. Letting E represent relevant evidence in the case, this requirement asserts that, within the P_M system, $P_M(H_i|E) > P_M(H_i^c|E)$, for every H_i . Since $P_M(H_i|E) + P_M(H_i^c|E) = 1.0$, $P_M(H_i|E)$ must be greater than 0.5. In addition, for the *overall* contention to be proved on the balance of probabilities we must have just a very few elements in the contention unless $P_M(H_i|E)$ is very large for all elements H_i . The reason is that the conjunction rule for P_M is multiplicative. If the H_i are independent conditional on E , then $P_M(H|E) = \prod_{i=1}^n P_M(H_i|E)$. If you multiply enough probabilities together (even large ones), sooner or later their product is less than 0.5. This same result will occur even if the H_i are conditionally noninde-

8. L. J. COHEN, *THE PROBABLE AND THE PROVABLE* 58 (1977).

pendent given E. Thus, a plaintiff in a complex civil suit involving many elements risks losing under a P_M interpretation even if he wins each element on the balance of probabilities. A constraint on complexity (number of elements in contention) is unknown in law and the author, therefore, concludes that the use of P_M is paradoxical in such cases.

The matter is resolved, says the author, when we construe complex civil suits in terms of P_I . Suppose $P_I(H_k, E)$ is the smallest value among the n values for plaintiff's case. By the conjunction rule for P_I :

$$P_I(H_1 \cap H_2 \cap \dots \cap H_n, E) = P_I(H_k, E).$$

Suppose further that $P_I(H_i, E) > P_I(H_i^c, E)$ for every element H_i ; i.e., plaintiff wins every point of his contention. The author says the jury can then infer that:

$$P_I(H_1 \cap H_2 \cap \dots \cap H_n, E) > P_I(H_1^c \cap H_2^c \cap \dots \cap H_n^c, E).$$

Now, the negation rule for P_I asserts that, if $P_I(H, E) > 0$ and $P_I(E^c) = 0$, then $P_I(H^c, E) = 0$ and, therefore, $P_I(H, E) > P_I(H^c, E)$. Thus, the inductive probability of plaintiff's entire contention is guaranteed to exceed that of defendant's contention if plaintiff wins each point on the balance of probabilities. Notice that there is no constraint on the number of elements in plaintiff's contention.⁹

Suppose, however, that plaintiff loses just one of his points; for element H_j , $P_I(H_j^c, E) > P_I(H_j, E)$. The conjunction rule then will assert that, for plaintiff's case, $P_I(H, E) = P_I(H_j, E)$. A further property of P_I (not included in the summary above) is called the *consequence* property. It asserts in this case that $P_I(H^c, E) \geq P_I(H_j^c, E)$. Thus, we have $P_I(H^c, E) \geq P_I(H_j^c, E) > P_I(H_j, E) = P_I(H, E)$. So plaintiff loses his overall contention on the balance of probabilities. In summary, in the P_I interpretation of the case, plaintiff wins the overall contention if and only if he proves all elements on the balance of probability.

2. Inference upon Inference

Considering civil cases again, the author believes P_M to

9. *Id.* at 58-59.

cause difficulties in cases involving "inference upon inference." Wigmore,¹⁰ referring to inference upon inference, believed that what he called "catenations" of circumstantial and testimonial evidence were common in most juridical inferences; for example, we seek to prove E from F and then prove H from E. Recent work¹¹ on the subject refers to this inference upon inference as "cascaded" or "multistage" inference. The author discusses how courts in civil cases normally require each level or catenation, prior to the final level, to rest upon proof beyond a reasonable doubt. Thus, in the above example we must prove E from F beyond a reasonable doubt in order to prove H from F on the balance of probabilities. In one possible formulation of this problem in terms of P_M the author shows how inferences about H on the balance of probabilities could arise when $P_M(E|F)$ is relatively small, surely not large enough in any case to satisfy the reasonable doubt requirement. His claim is that the multiplicative nature of the conjunction rule for P_M causes the difficulty. Because the operation of multiplication is commutative, large $P_M(E|F)$ times small $P_M(H|E \cap F)$ yields the same result as small $P_M(E|F)$ times large $P_M(H|E \cap F)$. Such "transitivity" is not altogether expected; the author claims that proof should depend upon what is probably inferable from known facts rather than upon what is certainly inferable from probable facts.

When we considered P_M conjunction-rule difficulties involving component issues in civil cases, we saw how P_M appeared quite severe in its effects upon proof on the balance of probabilities. For inference upon inference in civil cases P_M now seems liberal when evaluated against existing legal standards. The author argues that in the P_I system, establishing E on the premise F beyond a reasonable doubt establishes E as a known fact which provides a firm foundation for establishing H on the balance of probabilities. He further shows how the transitivity of proof mentioned above for P_M does not occur when the problem is construed in terms of P_I .

10. J. WIGMORE, *supra* note 1, at 13.

11. Schum, *Current Developments in Research on Cascaded Inference Processes*, in *COGNITIVE PROCESSES IN DECISION AND CHOICE BEHAVIOR* (T. Wallsten ed. 1979).

3. *Negation*

Again consider a civil suit in which plaintiff's overall contention H survives on the balance of probabilities; suppose $P_M(H|E) = 0.501$ and $P_M(H^c|E) = 0.499$. Cohen argues that the defendant in this case is entitled to conclude that the legal system has given itself a fairly wide margin for error. It has recognized, in effect, that there is a substantial likeliness that a losing case deserved to succeed; this, of course, is cold comfort to the losing defendant in our example. The fact that the negation rule for P_M is also complementational says that there is a determined amount of "case weight," and that this fixed amount of weight is allocated to plaintiff and defendant. The more one side gets the less the other side gets. One may well question why it should be true that increased support for plaintiff's contention means decreased support for defendant's contention.

The author argues that this P_M conception of balance of probabilities in civil case determination is not what is intended by our legal system. He says that civil litigation is a trial of case strength rather than a division of case merit and he believes that the P_I system reflects this interpretation. We recall that in the P_I system the negation rule is not complementational. In the case mentioned above, if we suppose that $P_I(E^c) = 0$, then $P_I(H, E) > 0$ implies that $P_I(H^c, E) = 0$. Also recall that in the P_I system "weight" of evidence refers to amount of evidence or the completeness of the coverage of relevant points. If we say that the evidence on balance favors plaintiff, $P_I(H, E) > 0$. Since it cannot at the same time favor defendant, $P_I(H^c, E) = 0$. Since plaintiff must win on every point of a contention, $P_I(H, E)$ will increase as more relevant points are considered provided that plaintiff's position is upheld on each point.

4. *Proof Beyond a Reasonable Doubt*

Now consider a criminal case in which H is the event that defendant is guilty of the charge. Letting E represent the evidence before the court, how large should $P_M(H|E)$ be in order to exceed the reasonable doubt standard? Suppose $P_M(H|E)$ is less than 1.0 by some small amount. On evidence E you conclude that H falls short of certainty by an acceptable amount and so

you conclude that H is established beyond a reasonable doubt. The author tells us that an interpretation of the reasonable doubt standard in terms of P_M is not appropriate. He argues that reasonable doubt exists when there is at least one specific reason for doubt and not simply that H falls short of certainty. Within his P_I system what is necessary to satisfy the reasonable doubt standard is a list of all relevant points that have to be established (*e.g.*, motive, means, opportunity, etc.) and satisfaction that the prosecution's case against defendant satisfies all of these points. Maximal $P_I(H, E)$ is achieved on the basis of your conception of the completeness of the coverage of evidence E on relevant matters. Beyond a reasonable doubt is achieved if you can think of no other specific reason for doubting the defendant's guilt.

5. *Criterion Difficulties*

Since, as we saw earlier, there are different possible interpretations of P_M , we have sooner or later to ask which one is appropriate for application to juridical issues. The author rules out classical probability for obvious reasons and rules out relative frequency for the reason that events in juridical matters are usually unique. Wager odds are ruled out for the reason that such odds involve nonprobabilistic issues such as the amount of money at stake. Finally, the author rules out a conception of P_M in terms of the "confirmation function," as proposed by Carnap,¹² on the grounds that there is an infinity of such functions and the choice among them is arbitrary. He argues that advocates of P_M must be able to supply a conception of P_M that fills the gap. He tells us that P_I does supply a conception of probability that is a natural representation of the manner in which most of us reason inductively in our everyday affairs. Since legal standards of proof and other prescriptions are meant for jurors who, it is assumed, will apply their ordinary reasoning ability to evidence at trial, the author concludes that P_I is a better description of what is meant by probability in existing legal standards and prescriptions.

12. R. CARNAP, LOGICAL FOUNDATIONS OF PROBABILITY (1950).

6. Difficulties Concerning Corroboration and Convergence

Corroborative testimonial evidence and converging circumstantial evidence are frequently encountered evidentiary patterns in juridical and other inferences. Testimonial corroboration occurs when two or more witnesses testify to the occurrence of the same event. Two or more items of circumstantial evidence are said to converge if, independently, they favor the same conclusion. Under suitably articulated conditions a conjunction of corroborative or a conjunction of convergent evidence raises the probability of conclusion H by an amount greater than that provided by any single item in the conjunction. The author presents several attempts to formalize these evidentiary patterns within the P_M system. He discusses an early attempt by Boole¹³ and a more recent attempt by Ekelöf;¹⁴ in addition the author provides a formalization of his own using P_M . The difficulty with any P_M analysis, according to the author, is that such analyses require the incorporation of prior probabilities of conclusions. He objects to any representation for the probative force of evidence which involves incorporation of prior probabilities of conclusions. Among the difficulties he cites of incorporating prior mathematical probabilities are:

(i) In civil cases if $P_M(H)$ is sufficiently strong, plaintiff could establish an overall case on the balance of probabilities even if one or more components of the contention are not established.

(ii) In the Anglo-American legal system, the accused is judged only upon evidence before the court; the accused does not come into court having a certain prior probability of guilt.

(iii) Within the P_M system, if the prior presumption of innocence is taken to mean very small prior probability of guilt, how this probability is to be determined is unclear.

According to Cohen, when corroboration and convergence are construed in terms of P_I they simply become different ways in which the inductive probability of conclusion H is increased by circumstances which are inductively relevant. The added advantage, he says, is that the inductivist analysis requires no

13. Boole, *On the Application of the Theory of Probabilities to the Question of the Combination of Testimonies or Judgments*, 21 *TRANSACTIONS OF THE ROYAL SOCIETY OF EDINBURGH* 597 (1857).

14. Ekelöf, *Free Evaluation of Evidence*, in 8 *SCANDINAVIAN STUDIES IN LAW* 1964, at 45 (F. Schmidt ed.).

assumption that the prior probability of any conclusion H be greater than zero. We may have $P_I(H) = 0$; then, evidence E_1 is presented which favors H and so $P_I(H, E_1) > 0$. If $P_I(E_1^c) = 0$, then $P_I(H^c, E_1) = 0$. Corroborative or convergent evidence E_2 is then provided for which $P_I(H, E_2) > 0$; this allows one to deduce that $P_I(H, E_1 \cap E_2) > P_I(H, E_1)$. Suitable independence conditions involving E_1 and E_2 can be articulated within the P_I system.

Two research topics of substantial current interest in experimental psychology concern subjective probability estimation and the revision of such estimations in light of additional evidence. A frequently employed research method is to present individuals with the necessary probabilistic ingredients of some problem and ask these persons to combine the ingredients in order to estimate the probability of some event specified in the problem. Following this, these subjective estimates are compared with corresponding calculations based upon formally coherent combinations of the problem ingredients. Of course, most of these calculations have been based upon the calculus of P_M . For example, individuals may be supplied with a set of priors $P_M(H_i)$ and likelihoods $P_M(E|H_i)$ for $i = 1, 2, \dots, n$ and asked to estimate $P_M(H_i|E)$ when E is known to have occurred. Their estimates are then compared with $P_M(H_i|E)$ calculated using Bayes's rule.

No one ever expects estimates of probability to be exactly equal to corresponding calculations based upon some formally coherent algorithm. Deviations occur, of course, but they are usually systematic. For example, under some conditions posterior probability estimates made by experimental subjects are typically more extreme than corresponding calculations prescribed by Bayes's rule. Under other conditions such estimates are typically conservative with respect to calculations using Bayes's rule. Another research method has involved inferring an individual's subjective probability for certain events based upon the person's stated preferences among alternative gambles based upon the occurrence or nonoccurrence of the events. A not unusual finding is that inferred subjective probabilities sometimes do not satisfy the complementation-negation rule for P_M .

The author has taken results from a series of experiments performed by Kahneman and Tversky¹⁵ involving various prob-

15. Kahneman & Tversky, *On the Psychology of Prediction*, 80 PSYCHOLOGICAL REV. 237 (1973).

ability estimation tasks. These researchers have, for a number of years, been interested in determining heuristic rules which individuals appear to use in estimating probabilities. These heuristic rules individuals appear to use are inferred from comparisons between probability estimates and P_M calculations. The author strongly objects to the statement in some of these studies that individuals are "incoherent" or are committing "fallacies" when their estimates do not agree with those prescribed by P_M . He proceeds to show how some of Kahneman and Tversky's results seem to be explainable under the assumption that subjects think according to P_I rather than according to P_M . This, he says, provides further argument in favor of his P_I system when applied to jurisprudence since legal prescriptions and standards involving probability are written so that they will relate to common or ordinary conceptions of probability. Such conceptions, the author believes, are inductive and not mathematical.

D. *Rebuttal Arguments in Defense of Pascal and His Heirs*

Before I proceed with arguments in defense of P_M applied to inferences at trial I must now tell you that I am a defendant in the case Cohen presents against P_M in such applications; in addition, I am not an attorney. I am told that the name given nonattorney defendants who plead their own cases is "guilty." This acknowledged, I will begin by telling you of the surprise I felt, upon reading the introduction of Cohen's work, to discover the level of his discontent concerning P_M applied in jurisprudence. The reason is that for the past few years I have been studying what the P_M system says are the necessary ingredients of various cascaded inference tasks. As I proceeded from simple to more complex cases I discovered a substantial level of agreement between the formalizations I was studying and corresponding prescriptions in the rules of evidence and in other related treatises. I was further encouraged by discovering that several evidence scholars having the background in law I do not have were arriving at similar conclusions about the efficacy of the essential approach I was taking.

These remarks do not form a preface to a violent attack upon Cohen's case against P_M . In fact, I agree with many of the concerns he exhibits about P_M when applied to court trials. Available juridical treatises contain an incredibly vast array of

inferential issues, more than enough to occupy the attentions of inference scholars over several lifetimes. It is obvious to me that Cohen and I have focused our attentions upon different aspects of juridical inference. We have examined different treatises or different parts of the same treatise and each of us has found a measure of support for his position. My belief is that no single formal system, including P_M or P_I , will form an adequate representation of every juridical inference task or will account for all of the assorted prescriptions made regarding inductive inference in our Anglo-American legal system. Following is a brief summary of the evidentiary issues I have examined which increased my confidence in the descriptive power of P_M applied in jurisprudence and which led to my initial surprise upon reading the early stages of Cohen's book.

Several years ago I became interested in what the P_M system says are the necessary ingredients of the task of determining the inferential value of testimonial evidence from witnesses whose credibility is less than perfect.¹⁶ Our formalizations showed, predictably, that the inferential value of such testimony depends upon the probative value of the event being reported and the credibility of the source. But these formalizations said much more. They prescribed the exact nature of the interaction between the rareness of an event and the credibility of the witness reporting this event in determining the probative value of the testimony from this witness. Such interaction was suspected but not successfully represented in the formalizations of La Place, Keynes, and others interested in juridical applications of P_M .¹⁷ Studying testimony from several witnesses, where such testimony is either corroborative or contradictory, our resulting formalizations show why the aggregate credibility rather than the number of witnesses is important in determining the probative value of such evidence.¹⁸ This corresponds with a legal pre-

16. Schum & Du Charme, *Comments on the Relationship Between the Impact and the Reliability of Evidence*, 6 ORGANIZATIONAL BEHAVIOR AND HUMAN PERFORMANCE 111 (1971).

17. J. KEYNES, *supra* note 4; P. LA PLACE, A PHILOSOPHICAL ESSAY ON PROBABILITIES (Dover ed. 1951).

18. Schum & Kelly, *A Problem in Cascaded Inference: Determining the Inferential Impact of Confirming and Conflicting Reports from Several Unreliable Sources*, 10 ORGANIZATIONAL BEHAVIOR AND HUMAN PERFORMANCE 404-23 (1973).

cription mentioned by Cleary¹⁹ and by Wigmore.²⁰ We also discovered that our formalizations for the probative value of testimonial evidence included ingredients necessary for incorporating the major legal grounds for impeaching the credibility of a witness and that our formal process was adequate in accounting for what the rules of evidence say are the essential components of hearsay evidence.²¹

Another formal investigation concerned what has been termed the process of "connecting up" the evidence or evaluating a current item in light of previously given items.²² Resulting formalizations show that the process of evaluating a current item of evidence against a background of prior evidence is akin to color, brightness, and other contrast processes in sensory perception.²³ In making a connection between inductive inference and sensory processes, we noted that it was Hume who said:

All probable reasoning is nothing but a species of sensation . . .
when I give the preference to one set of arguments over
another, I do nothing but decide from my feeling concerning
the superiority of their influence.²⁴

These formalizations for the process of connecting up evidence also show conditions, such as those the courts recognize, under which an item of evidence having no probative value on its own is, nevertheless, deemed relevant because it conditions opinion about other evidence which is relevant. In very recent investigations we have examined various additional evidence subtleties such as the locus and extent of redundancy in cumulative and corroborative evidence,²⁵ and the joint role of bias-related and observational sensitivity-related factors which determine the credibility of a witness and, ultimately, the probative value of

19. MCCORMICK ON EVIDENCE (2d ed.).

20. J. WIGMORE, *supra* note 1, at 318.

21. Schum, *On the Behavioral Richness of Cascaded Inference Models: Examples in Jurisprudence*, in COGNITIVE THEORY 149 (N. Castellan ed.).

22. R. LEMPERT & S. SALTZBURG, A MODERN APPROACH TO EVIDENCE 143-44 (1977).

23. Schum, *Contrast Effects in Inference: On the Conditioning of Current Evidence by Prior Evidence*, 18 ORGANIZATIONAL BEHAVIOR AND HUMAN PERFORMANCE 217 (1977).

24. D. HUME, TREATISE ON HUMAN NATURE, Book I, Part III, Section VIII (1881).

25. D. Schum, *On Factors Which Influence the Redundancy of Cumulative and Corroborative Evidence* (Dept. of Psychology, Rice University, Report No. 79-02).

what the witness reports.²⁶ In none of these investigations could we discover corresponding juridical prescriptions to which our formalizations seemed uncongenial.

As I read further in Cohen's book I was very impressed by the elegant way in which he develops the essentials of his P_I system. I was also troubled by the paradoxes he mentions concerning application of P_M in jurisprudence. I have no argument against the legitimacy of P_I as an appropriate index of uncertainty in inductive inference tasks. The author correctly points out that some over-zealous mathematicists frequently refer to P_M as "the" formal system of probability. P_M is, in fact, just one conceptual model for the articulation and manipulation of measures of uncertainty. Following are some general concerns I have about the author's work as it is presented in *The Probable and the Provable*.

1. On the "Weight" of Evidence

My first concern has to do with the concept of relevancy and the establishment of the probative weight of evidence. In chapter 10 on the topic of corroboration and convergence, the author argues against any measure of the probative weight of an item of evidence in terms of the difference between the posterior probability of a conclusion, given this item, and the prior probability of the conclusion. However, Federal Rule of Evidence 401 defines relevant evidence as follows:

"Relevant evidence" means evidence having any tendency to make the existence of any fact that is of consequence to the determination of the action more probable or less probable than it would be without the evidence.²⁷

This definition seems to specify a prior-posterior comparison of the exact sort the author rejects. Within P_M it seems natural to construe the probative weight of evidence in terms of likelihood ratio, which is one appropriate and desirable measure of the change from prior to posterior opinion about the likeliness of a fact-in-issue. In discussing the properties of P_M I listed

26. D. Schum, *Sorting out the Effects of Witness Sensitivity and Response Criterion Placement Upon the Inferential Value of Testimonial Evidence* (Dept. of Psychology, Rice University, Report No. 79-01).

27. FED. R. EVID. 401.

Bayes's rule as Property (vii). From this property it is easily seen that the likelihood ratio for evidence E is equivalent to the ratio between posterior odds and prior odds of H to H^c ; thus it measures the change in your opinion about the relative likeliness of H to H^c , given E . I am not alone in this interpretation of rule 401. Others with the background in law I lack have reached the same conclusion.²⁸

The prior probability of any H is, of course, always relative to the evidence you have. Let $F = (E_1 \Omega E_2 \Omega \dots \Omega E_n)$ represent the first n items of evidence you receive; then, $P_M(H|F)$ represents your prior probability of H before you receive the next item E_{n+1} . The likelihood ratio for E_{n+1} is

$$\frac{P_M(E_{n+1}|H)}{P_M(E_{n+1}|H^c)} = \frac{P_M(H|F \Omega E_{n+1})}{P_M(H^c|F \Omega E_{n+1})} \bigg/ \frac{P_M(H|F)}{P_M(H^c|F)};$$

this measures the change in relative likeliness of H to H^c from prior to posterior upon knowledge of E_{n+1} . Since rule 401 says nothing about how much evidence has been presented, it apparently recognizes that you will have some opinion about the likeliness of H before *any* evidence is presented at trial. In this case, let $P_M(H)$ represent your *initial* prior probability of H . Then, $P_M(H|E_1)$ represents the posterior probability of H after the first item of evidence. If this were not the case, we could never establish the relevancy of the first item of evidence, thus, never get the trial started. Both the P_M and P_I systems have unconditional, prior, or monadic probabilities; that is, they both acknowledge that your opinion about the likeliness of H has to have some initial state if we can say it has been revised on the basis of evidence. Neither P_M or P_I assume that you begin an inference task with a mental *tabula rasa*. The essential difference between P_M and P_I concerns how these and other probabilities are interpreted and how they can be manipulated.

As mentioned earlier, the author says that $P_I(H)$ represents "nature's potential for bringing about H ." Presumably then, $P_I(H, E)$ represents nature's potential for bringing about H , on the premise that E occurred. The difficulty is that, since P_I measures are ordinal in nature, $P_I(H)$ and $P_I(H, E)$ can never

28. Kaplan, *Decision Theory and the Fact-Finding Process*, 20 STAN. L. REV. 1065 (1968); Lempert, *Modeling Relevance*, 75 MICH. L. REV. 1021 (1977). See also R. LEMPert & S. SALTZBURG, *supra* note 22, at 148.

be algebraically compared in attempts to measure the change from prior to posterior likelihood of H as a result of E . However, within the P_I system, we can say that there is more evidence for H or that there is greater evidential coverage for H now that you know E . Thus it appears that prior-posterior comparisons are natural within both P_I and P_M and both appear consistent with rule 401. I will again discuss P_M measures of probative value when I consider the specific paradoxes the author has mentioned.

2. *Difficulties in Posing Inferential Problems*

In discussing the paradoxes of P_M and in making other comparisons between P_I and P_M the author cites various P_M formalizations which he then proceeds to criticize in various ways; in one or two cases he derives these P_M formalizations himself. In several cases, notably concerning examples of inference upon inference and corroborative and convergent evidence, I believe the presented formalizations do not contain all necessary probabilistic ingredients. Part of the problem is that the presented formalizations involving testimonial evidence require but do not contain distinctions of the following sort. Let E = the event that the traffic light at the scene of the accident was on green, and let E_i^* = the event that witness W_i testifies that the traffic light at the scene of the accident was on green. The events E and E_i^* are not equivalent events since, of course, we may have E_i^* when E is true or E_i^* when E^c is true. As a juror confronted with testimonial evidence, your conclusion about H is conditioned by E_i^* and not by E . A vague definition of a conditioning event such as F = "the event that the witness testified correctly" can contain no useful probative information since F does not tell us to what event the testimony refers. Such information is crucial in establishing the credibility-related ingredients of the formalization for the probative weight of testimonial evidence. In one case involving corroborative and convergent evidence a formalization was cited which was formally incoherent within the P_M system.²⁹

In any formal system it is altogether crucial that problems be posed adequately. In the case of inductive inference problems

29. L. J. COHEN, *supra* note 8, at 99-100.

this involves careful determination of what constitutes the appropriate conditioning event (it is not always obvious) and it involves articulation of appropriate conditional independence considerations. None of the P_M formalizations discussed by the author involve likelihood ratios. Certain features of the P_M system are not evident unless formalizations involving probative weight are expressed in this fashion. In short, I believe the P_M system can be shown to much better advantage than it is in Cohen's work.

3. *Subjective Probability: The Case of the Missing Criterion*

In discussing various interpretations of P_M at the beginning of his book, the author mentions the subjective or personal interpretation of probability. Later, in discussing the "paradox" involving a choice of P_M interpretation for juridical application, he avoids mention of personal or subjective probability as one possible choice. We recall that he does rule out classical, relative frequency, and confirmation-function interpretations. Since the author does not mention the subjective interpretation, presumably he does not rule it out. On the other hand, he may simply be treating the subjective interpretation with disdain. After all, in his early discussion of the subjective interpretation of probability he says that "few researchers in the natural or social sciences have in fact adopted this personalistic approach."³⁰

My view on these matters is somewhat different; within P_M I believe that the only extant interpretation which does merit serious consideration in juridical inference is the personalistic or subjective approach in which, subject to some coherence constraints, a subjective probability measures your strength of belief in the occurrence of some event. Other individuals within jurisprudence apparently share this view as well. Wigmore,³¹ for one, defines a measure of proof to be "the degree of strength of belief of a fact-in-issue produced by evidence on the mind of the jury." It is interesting to note that contemporary scholars in probability theory define a subjective or personal probability

30. *Id.* at 27.

31. 9 J. WIGMORE, EVIDENCE, § 2498a (3d ed.).

in Wigmore's terms. DeFinetti³² defines a personal probability to be "a measure of the degree of belief of a given subject in the occurrence of an event." Cohen might be tempted to argue that Wigmore's definition in fact corresponds to his conception of probability as a degree of provability. This would be fine and, if so, I would then see little difference between P_I and the subjective interpretation of P_M except in matters of scale properties. Finally, I cannot agree that the subjective interpretation of probability appeals to few researchers in the natural and social sciences. I find it hard to believe that the author is innocent of the prevalence of the subjective interpretation of probability in decision-theoretic areas of economics, business, medicine, and psychology. In these and other areas many researchers would prefer to use Bayesian statistical methods for analysis and interpretation of data but do not because they also wish to have their papers accepted by journal editors who, all too frequently, adhere to classical statistical approaches based upon a relative-frequency interpretation of probability and reject other approaches, not always for informed reasons.

Uncertainty about fact-in-issue H need not be expressed in terms of probability. For example, posterior odds $P_M(H|E) / P_M(H^c|E)$ expresses the likeliness of H relative to the likeliness of H^c , on the premise that E occurred; likelihood ratio $P_M(E|H) / P_M(E|H^c)$ expresses the likeliness of evidence E under H relative to the likeliness of E under H^c . The author correctly rejects wager or betting odds as a representation for P_M in juridical applications since wager odds (such as those supplied by your bookmaker) depend upon other factors such as the size of the stakes involved and the number of wagers placed. However, wager odds and either posterior or prior odds are not the same. It is easily shown that they are mathematically equivalent only for fair bets. Thus, posterior or prior odds can be expressions of uncertainty and nothing else. Expressions of uncertainty in odds and likelihood ratios have the advantage that they are not bounded above as is the probability measure. I quickly note, however, that this change of scale does not remove the necessity for adherence to the P_M axioms as far as the ingredients of these ratios are concerned.

In much research on human inference behavior as well as in a variety of applied tasks, individuals express relative uncer-

32. B. DEFINETTI, *PROBABILITY, INDUCTION, AND STATISTICS* 731 (1972).

tainty in terms of odds and likelihood ratios.³³ There is an important issue here. The P_I system credits individuals with being able to make only weak order judgments regarding uncertainty (*i.e.*, "greater than," "less than," or "equal to" judgments). The subjective interpretation of probability allows you to credit individuals with being able to make ratio judgments of uncertainty. Is this confidence justified? As I mentioned earlier, forming ratios on some scale assumes a zero point and a unit on the scale; do subjective scales have such properties? We are here confronted by one of the oldest and most perplexing problems in all of psychology. Do the subjective continua of brightness, loudness, heaviness, or indeed likeliness, have equal units up and down their scales? Perhaps no one will ever know; at least there is no obvious methodology for making such a determination.

However, one thing clear is that individuals in a variety of behavioral tasks make ratio judgments naturally and coherently; there are several methods for evaluating the internal coherency of such judgments.³⁴ There is, in fact, good reason for confidence in human ability to make ratio judgments. The necessity for preserving ratios of stimulus magnitude is readily apparent and is verified by an assortment of commonly observed perceptual invariances.³⁵ Unless our sensory-perceptual apparatus had finely tuned mechanisms for preserving ratio information, speech could only be understood at one level of volume and a picture or a face would only be recognizable at one level of illumination. If you are wondering what all this sensory-perceptual business has to do with probability and juridical inferences, perhaps you have forgotten Hume's argument (cited above) in which he says that all probable reasoning is a species of sensation. I, for one, have taken this analogy very seriously in efforts to understand complex inference.³⁶

In both the P_M and P_I systems probability is viewed as a quantitative rather than a qualitative property. One difference between the two systems is that P_I assumes ordinal judgments only whereas subjective interpretations of P_M allow for ratio

33. See, *e.g.*, Slovic & Lichtenstein, *Comparison of Bayesian and Regression Approaches to the Study of Information Processing in Judgment*, 6 ORGANIZATIONAL BEHAVIOR AND HUMAN PERFORMANCE 649 (1971).

34. L. MARKS, *SENSORY PROCESSES* (1974).

35. S. STEVENS, *PSYCHOPHYSICS* (1975).

36. Schum, *supra* note 23.

judgments of the sort we have discussed. Under this subjectivist view, when a person tells you H is ten times more probable than H^c , given evidence E , this person probably means what he says and is likely to give the same response again under similar circumstances. It is interesting to note that research indicates that individuals' estimates come closer to P_M calculations when they respond in odds rather than in probabilities.³⁷ To reject human ability to form ratios is to reject the results of countless experiments in which such judgments were made naturally and consistently.

Following are specific comments I have about the paradoxes the author claims to exist when P_M is applied to juridical inference.

a. *Conjunction rule difficulties in civil suits.* Since the law requires plaintiff to prove each element or point H_i on the balance of probabilities, we must have $P_M(H_i|E) > P_M(H_i^c|E)$ if H_i is independent of other elements; this requires $P_M(H_i|E) > 0.5$ as we have seen. If the elements are not independent we may have, for example, the requirement that $P_M(H_2|H_1 \cap E) > P_M(H_2^c|H_1 \cap E)$; this means that $P_M(H_2|H_1 \cap E) > 0.5$. If the proving of each point constitutes a separate inference problem, then there is no difficulty for P_M . If, in addition, as the author argues, plaintiff must prove his overall contention $H = (H_1 \cap H_2 \cap \dots \cap H_n)$ on the balance of probabilities, then there are difficulties for P_M . In this case we must have $P_M(H_1 \cup H_2 \cup \dots \cup H_n) > P_M(H_1^c \cap H_2^c \cap \dots \cap H_n^c)$. It is a relatively easy matter to establish that, under this additional requirement, the product $P_M(H_1|E) \cdot P_M(H_2|H_1 \cap E) \cdot \dots \cdot P_M(H_n|H_{n-1} \cap H_{n-2} \cap \dots \cap H_2 \cap H_1 \cap E)$ must be greater than 0.5. If you agree with the author that both requirements must be met, then P_M produces the paradoxical results the author claims. Some may wish to argue, however, that the additional requirement of proof of the overall contention constitutes inferential double jeopardy. Having won each individual contest to the satisfaction of the law, why should the defendant now be required to show that he won all of them? Cohen says this is closing one's eyes to facts one does not like.

b. *Inference upon inference.* Suppose you must prove E from F and then prove H from E . In civil cases the author tells

37. Phillips & Edwards, *Conservatism in a Simple Probability Inference Task*, 72 J. EXPERIMENTAL PSYCH. 346, 352-54 (1966).

us that a frequent constraint is that, though H may be proved from E on balance of probabilities, E must be proved from F beyond reasonable doubt. The author, speaking normatively, says this corresponds with expectation since proof should depend upon what is probably inferable from known facts rather than what is certainly inferable from probable facts. These are extremes, of course; what about the commonly occurring instance in which proof must depend upon what is probably inferable from probable facts? This seems to characterize many situations in civil and criminal trials in which inference rests upon catenations of circumstantial events and in which information about these events comes from witnesses who are not infallible observers and who are subject to all manner of biasing and other motivational influences.

Consider a simple inference upon inference involving testimonial evidence where the occurrence of E is to be proved from testimony E_i^* from witness W_i and H is to be proved from E . Under the above-mentioned constraint, though H be provable from E on balance of probability, E must be proved from E_i^* beyond reasonable doubt. Perhaps in those instances in which this constraint is not enforced the court has recognized something which well-posed P_M formalizations make clear: an already weak linkage between $\bar{E}$ and H cannot be much disturbed by a less than perfect linkage between E_i^* and E nor can a strong linkage between E and H be preserved by a weak linkage between E_i^* and E . The observed transitivity of the P_M formulations simply asserts that in establishing the probative value of testimony there can be trade-offs between the probative value of the event being reported and the credibility of the source reporting the occurrence of this event.

c. *Negation.* Imagine a civil suit tried according to P_M in which plaintiff proves all points on the balance of probability and proves the overall contention H on balance of probability. Suppose $P_M(H|E) = 0.501$; then, as P_M requires, $P_M(H^c|E) = 0.499$. As the author argues, defendant can feel cheated because the court apparently recognizes that there is a substantial probability of his contention H^c on the evidence presented. Now let us try another case, this time according to P_I , in which plaintiff loses one point H_i on balance of probabilities. When plaintiff asks what the value of $P_I(H_i, E)$ is, he cannot believe his ears when told that it is zero. Surely, he says, there has been suppression of uncertainty somewhere in the process. We argue that he

does not understand inductive probability. Apart from the fact that no conception of probability is likely to make a loser any happier, the negation principle for P_I operates with a harshness and discontinuity which many persons will not find characteristic of their own inductive reasoning processes. However, the author tells us that his claim is the modest one that some people use the P_I concept, particularly in law courts.³⁸

d. *Beyond a reasonable doubt.* I freely admit to some discomfort about the nonspecific nature of this forensic standard of proof when matters are construed in terms of the P_M system. The author's P_I system is much more specific regarding this standard of proof but I believe there is still reason for discomfort. Within P_I , proof beyond a reasonable doubt means proof at a level of inductive certainty; *i.e.*, when there is no specific reason for doubting the truth of a conclusion. Maximum $P_I(H, E)$ is obtained when evidence E includes all relevant points to be established together with satisfaction that the prosecution has proven these points. I, for one, have difficulty imagining a situation in which *all* relevant points have been listed, let alone established in fact.

Consideration of witness credibility is of obvious importance here but I find the author to be rather vague about the formal connection between the credibility of a witness and the probative value of the testimony given by the witness. He regards the probability that the witness is telling the truth as an inductive probability whose value depends upon the grade of inductive support for the prediction that the witness will give credible testimony, given what you know about the witness. I could not discover how credibility-related information is combined, if at all, with information about the probative value of the event the witness reports in order to establish the probative value of the testimony. In part, this is due to the fact that the author does not distinguish between the occurrence of event E and the testimony E_i^* of this event given by a witness.

I have already spoken my piece on the "criterion" difficulties mentioned by the author. The next difficulty on his list concerns corroborative and converging evidence. The author's main point here concerned his arguments against prior-posterior analyses in establishing the weight of evidence. I have also discussed this

38. L. J. COHEN, *supra* note 8, at 118.

matter above and have shown how such a prior-posterior analysis is consistent with rule 401 on relevancy.

IV. IN CONCLUSION

I have told you that I am a defendant in the case Cohen presents against the interpretation of legal standards of proof and of other evidentiary issues in terms of Pascalian or mathematical probability. A defendant's praise for the work of a prosecuting attorney is perhaps rare enough to be noteworthy and I do indeed offer my praise for Cohen's work. At least the length of this review should suggest how seriously I took his arguments. The system of inductive probability he offers does explain certain paradoxes with which one is confronted when one tries to apply mathematical probabilities to juridical inference. The mathematical probability system is now very rich and some very difficult inferential problems can find formal representation within this system. However, the author believes that the price to be paid for this richness is too high. After you read his work you may or may not agree about the extravagance of the price. However, this book is guaranteed to deflate the hardened mathematicist who may perceive that mathematical probability is the only system within which canons for coherent probabilistic inference can be found.

Not all of this book can be read easily without some background in formal logic. However, the parts of the book that concern juridical inference can, I believe, be read quite easily without this background. This book ought to be read by other persons whose research interests relate to juridical inference. The author discusses how his system of inductive probabilities seems to account for some of the results found by experimental psychologists when they study human behavior in probabilistic inference tasks. In summary, I found *The Probable and the Provable* a profound and stimulating work of scholarship, and I close my comments with applause for its author.