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Sharp Lines
and Sliding Scales in Tax Law

EDWARD FOX AND JACOB GOLDIN*

INTRODUCTION

The law is full of sharp lines, where small changes in one's circumstances lead to significant changes in legal treatment. Tax law is no exception; someone who falls just to one side of a line may be taxed quite differently from someone else who falls just to the other side, even when the two situations are otherwise quite similar. Consider the following examples:

- The sale of a capital asset held for 366 days is eligible for the preferential rate on capital gains, but the sale of the same asset held for 364 days is not.¹
- Taxpayers who contribute appreciated property to a corporation in exchange for stock do not recognize gain or loss if they own at least 80% of the corporation after the transaction; if the taxpayers own only 79%, they recognize the gain.²
- An employer with fifty-one employees is required to offer health insurance coverage to its employees or face a penalty; an employer with forty-nine employees is not.³

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¹ IRC §§ 1(h), 1222.
² IRC §§ 351, 368(c).
³ IRC § 4980H(c)(2).
• A child born on New Year's Eve can qualify the parent for the full year's Child Tax Credit; a child born the next day yields no tax benefits for the prior year.4
• For certain states, an individual who spends 183 days of the year in the state is considered a resident, whereas an individual who spends 182 days in the state is not.5

In many cases, a sharp line can be smoothed out by replacing it with a sliding scale. Whereas classification under a sharp line is all-or-nothing, a sliding scale classifies individuals in proportion to where they fall along some continuum. Hence, under a sliding scale, small changes to a situation lead to small changes in the legal outcome. To illustrate, a sliding scale might allow taxpayers to claim the Child Tax Credit in proportion to the share of the year that elapses after the child's birth. Similarly, a sliding scale for long-term capital gains might gradually phase in the preferential tax rate based on the holding period—say, between six and eighteen months.6

Historically, a significant disadvantage of sliding scales compared to sharp lines was their complexity; in an age when taxpayers or their accountants calculated taxes by paper and pencil, the extra work required to apply a sliding scale would usually have dwarfed the benefits—especially when applied to the tax rules governing individuals. Today, however, the vast majority of tax returns are prepared electronically. As a result, complexity concerns are no longer as important a barrier to sliding scales as they once were.

Motivated by this development, we study the policy choice between sharp lines and sliding scales in tax law. We compare the two approaches along a range of dimensions, including efficiency, complexity, and fairness. Our main conclusion is that sliding scales deserve consideration in many of the settings in which the tax law currently employs a sharp line. For many of these classifications, there is a strong case for converting the sharp line into a sliding scale. We illustrate the principles that emerge from our analysis by applying them to several real-world examples: child tax benefits, taxation of capital gains, conventions for depreciable property, state residency rules, and treatment of debt and equity instruments.

We begin by comparing the efficiency properties of sharp lines and sliding scales. Drawing on a simple model of taxpayer behavior, we find that the relative efficiency of the two approaches depends on the pattern of choices that taxpayers would make in the absence of tax

4 IRC §§ 24, 152.
5 See, e.g., Minn. Stat. § 290.01, subd. 7(b) (2019).
6 As we discuss below, each of the sharp lines listed in the prior paragraph could be replaced by a sliding scale.
considerations. Intuitively, a sharp line only affects those taxpayers whose behavior would put them close to the line, but it can significantly distort the behavior of this group. In contrast, a sliding scale affects more taxpayers, but the magnitude of the distortion it generates is smaller. We conduct simulations to explore these trade-offs under a range of assumptions about taxpayer preferences and behavior. We find that the efficiency rationale for a sharp line is strongest when few taxpayers are close to where it would be drawn. When this is not the case, we find that sliding scales tend to be more efficient.

We next consider the relative complexity of sharp lines and sliding scales. Although sliding scales make it harder for taxpayers to compute their precise tax liability, we argue that this consideration is rarely compelling given the large share of taxpayers whose returns are prepared electronically. On the other hand, sliding scales can require more information about taxpayers than sharp lines, and this form of complexity affects even those using assisted preparation methods. We argue that the force of this concern varies greatly by setting. With respect to determining state residency, for example, a sliding scale might entail large costs if it requires tracking the precise number of days one spends in each state. In contrast, using a sliding scale for tax benefits based on a child's birth date would not require taxpayers to keep track of any new information.

We then explore how sliding scales and sharp lines interact with fairness considerations and the objectives of specific provisions of the tax code. Although it is difficult to draw broad conclusions here, we make two general observations. First, sliding scales tend to be preferable when a provision's objective applies similarly to taxpayers who would fall on opposite sides of a sharp line. For example, if child tax benefits are motivated by households with children having lower ability to pay, it follows that taxpayers with children born on December 31 and January 1 should be taxed similarly, because both support a child for similar amounts of time. Second, where the goal of a provision is to shape taxpayer behavior, our conclusions are the mirror image of those that apply in the efficiency analysis. That is, sharp lines are more effective than sliding scales at shaping behavior for taxpayers close to

7 For a related argument with examples drawn from tax, see Bradley T. Borden, Quantitative Model for Measuring Line-Drawing Inequity, 98 Iowa L. Rev. 971 (2013) (describing how bifurcation can reduce the inequitable all-or-nothing treatment that stems from a sharp line). More generally, see Larry Alexander, Scalar Properties, Binary Judgments, 25 J. of Applied Phil. 85 (2008); Adam J. Kolber, Smooth and Bumpy Laws, 102 Cal. L. Rev. 655 (2014); Lee Anne Fennell, Slices and Lumps: Division and Aggregation in Law and Life (2019). Of course, as we discuss below, the converse is also true: A sharp line tends to be preferable if the objective underlying the policy is discontinuous at the location where the line is drawn.
where the line is drawn and less effective for taxpayers far from the line.

We also consider the choice between sharp lines and sliding scales from the perspective of the tax administrator. Each approach poses its own enforcement challenges, which, as with the other considerations we discuss, varies significantly by context. To illustrate, it may be that a smaller share of tax returns report the correct tax liability under a sliding scale, but that the magnitude of the reporting mistakes is larger under a sharp line.

Finally, sliding scales and sharp lines differ in the degree to which they induce taxpayers to engage in tax planning. Depending on the setting, this difference can either exacerbate or alleviate the efficiency costs of a tax. We describe how switching from a sharp line to a sliding scale can alter both the costs and benefits of tax planning, with the net effect varying from case to case. The net effect of tax planning on social welfare is itself also ambiguous: Taxpayers are privately worse off from failing to fully consider the tax when deciding on their behavior, but society as a whole may benefit from more tax revenue or lower tax rates.

Perhaps because the widespread use of sliding scales in tax has only recently become practical, the policy choice between them and sharp lines has not received much attention. For example, an influential set of analyses studies the question of where to draw the (sharp) line that divides two legal categories, but does not consider the antecedent question of whether a sharp line is desirable in the first place. Similarly, the large literature that analyzes the choice between rules and standards is focused on the precision with which a (sharp) line between legal classifications is set out and does not consider the desirability of a sliding scale.

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10 In our nomenclature, both rules and standards are typically assumed to be sharp lines, in that the consequence of crossing the line—whether its precise location is known or not—results in a discontinuous change in legal treatment. In our analysis, we will mostly focus on the choice between sharp line rules and sliding scale rules, but one can also imagine choosing between sharp line standards and sliding scale standards, such as the choice between a pure comparative negligence regime and a contributory negligence regime for tort liability. We consider the special issues that arise for sliding scale and sharp line standards in Part VII.A.
Notwithstanding the lack of a general treatment, others have considered specific applications of our question, such as in discussions about the taxation of hybrid financial instruments, the definition of a capital asset, the distinction between debt and equity, and the apportionment of tax liability across jurisdictions. Our hope is that approaching the choice between sharp lines and sliding scales at a higher level of abstraction will produce lessons that complement the insights obtained in these specific domains.

Another related issue that has received attention is how tax liability should vary by income, and in particular, the manner in which various provisions should be subject to income phase-ins or phase-outs. Here, the conventional wisdom is that sharp lines—or, "cliff effects" as they're often referred to when tax liability depends on income—are undesirable, and should be avoided whenever possible. Although we don't focus on income-based classifications, a benefit of our framework is that it is general enough to compare sliding scales and sharp lines in both income and non-income settings. In the case of classifications based on income, our results largely support the conventional view that phase-ins are preferable to sharp lines. But for non-income classifications, our analysis suggests that the proper approach can vary across contexts.

Finally, outside of tax, prior work has noted the distinction between sharp lines and sliding scales in diverse areas of law, especially in the

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15 For an insightful discussion of this topic, see Manoj Viswanathan, The Hidden Costs of Cliff Effects in the Internal Revenue Code, 164 U. Penn. L. Rev. 931 (2016). Although Viswanathan's focus is on income-based cliff effects, he claims that "[c]liff effects in the Internal Revenue Code based on metrics other than income can be reconciled with notions of equity and efficiency." See id. at 954. We add nuance to that conclusion by highlighting conditions in which sliding scales are actually preferable to sharp lines on both equity and efficiency grounds, including for classifications based on characteristics other than income.
16 Id.; see also Joel Slemrod, Buenas Notches: Lines and Notches in Tax System Design, 11 eJournal Tax Res. 259, 275 (2013) (citing literature and arguing that in the absence of administrative costs, a fully flexible optimal tax system would never have income-based sharp lines).
17 See, e.g., John E. Coons, Approaches to Court Imposed Compromise—The Uses of Doubt and Reason, 58 Nw. U. L. Rev. 750, 758-59 (1963) (focusing on courts imposing...
domains of tort and criminal law, as well as in moral theory more broadly. To our knowledge, however, we are the first to develop a general framework for comparing the behavioral effects of sharp lines and sliding scales. Indeed, our model can inform the choice between these policies in tax and nontax settings alike.

In the public finance literature, economists have studied the behavioral effects of sharp lines and sliding scales, both as separate instruments and more generally as a question of optimal taxation. As with legal scholarship, the public finance literature tends to focus on the properties of sharp lines, or "notches" as they're often referred to in this field, in the context of how tax liability depends on income. Separately, a large and growing empirical literature within economics exploits policy notches to estimate various determinants of individual behavior.

The remainder of the Article proceeds as follows. Part I presents a stylized model to compare the effects of sharp lines and sliding scales on taxpayer behavior. Part II focuses on the efficiency trade-offs between the two types of policies. Part III considers issues relating to compromise outcomes in the case of a doctrinal or evidentiary tie); Gideon Parchomovsky, Peter Siegelman & Steven Thel, Of Equal Wrongs and Half Rights, 82 N.Y.U. L. Rev. 738 (2007); Lee Anne Fennell, Lumpy Property, 160 U. Pa. L. Rev. 1955 (2012). For more general treatments of this issue, see Leo Katz, Why the Law Is So Perverse 139-81 (2011) (defending the existence of law's "either-or" treatment partly on the basis of social choice theory); Kolber, note 7 (developing a taxonomy of the relationship between legal inputs and outputs and considering the desirability of the various approaches across legal contexts, including a brief discussion of tax law).

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19 Alexander, note 7; see also Derek Parfit, Reasons and Persons 199-350 (1984) (discussing the continuous nature of changes in personal identity over time).

20 Of course, the relevant normative considerations are likely to differ in nontax settings. Unlike other areas of law, for example, taxes are typically designed to avoid affecting how individuals behave.

21 The classic public finance article laying out the behavioral and efficiency trade-offs between linear tax incentives and sharp lines is Alan S. Blinder & Harvey S. Rosen, Notches, 75 Am. Econ. Rev. 736 (1985). In Part II, we develop these principles in the context of the classification challenges that frequently arise in tax law.

22 For a summary of this literature, as well as an exception to the focus on income-based notches, see Slemrod, note 16.

23 For an overview, see Henrik Jacobsen Kleven, Bunching, 8 Ann. Rev. of Econ. 435 (2016).
complexity. Part IV considers how well sharp lines and sliding scales effectuate the policy goals underlying specific tax provisions. Part V considers tax administration. Part VI considers implications for tax planning. Part VII focuses on several special cases, relating to standards versus rules; classifications based on taxpayer intent; democratic legitimacy; classifications based on income; and multifactor tests. Part VIII illustrates our analysis with examples relating to child tax benefits, the capital gains holding period, conventions for depreciable property placed in service, state residency tests, and the treatment of debt versus equity instruments. Part IX concludes.

I. Framework

This Part introduces a simple model of taxpayer behavior, discusses its key assumptions, and considers the range of settings to which our analysis applies.

A frequent task of legal doctrine is to classify situations into one category or another, such as whether a party to a contract has breached its obligation, whether some evidence is admissible into court, or whether a defendant is guilty of a charged offense. In tax, such classifications are used for the purpose of assigning tax liability to taxpayers. If one thinks of the tax law as a function that maps taxpayers' characteristics into tax liability, legal classifications represent intermediate steps. Depending on how a taxpayer is classified, both the amount of tax owed and the relation between tax and income can differ. For example, a taxpayer classified as head of household faces different tax rates than someone classified as single. A business classified as a partnership faces a different tax regime than one classified as an S corporation.

In reality, one's tax liability is a function of many characteristics—such as age, marital status, and income amount and source. To keep things simple, though, we'll generally restrict our focus to settings in which tax liability varies based on a single characteristic (x), holding fixed all of the other factors upon which tax liability depends. Because \( x \) determines which legal category the taxpayer falls into, we refer to it as the classification variable. We assume that \( x \) can take on values between 0 and 1 (inclusive).

Our focus is on settings in which the tax law seeks to differentiate taxpayers based on their value of \( x \). In particular, we assume that the law treats taxpayers with values of \( x \) at 0 differently from taxpayers with values of \( x \) at 1. Thus, if we let \( t(x) \) denote tax liability as a

\[24\] We use "characteristic" to denote any feature of taxpayers or their situations that could potentially affect their tax liability.
function of $x$, $t(0) \neq t(1)$, where $t(0)$ and $t(1)$ denote liability under the $x = 0$ and $x = 1$ tax regimes, respectively. Hence, $x$ might represent any characteristics of a taxpayer that can be represented using the zero-to-one scale and for which the tax law distinguishes between taxpayers falling at opposite ends of the continuum. For example, $x$ might represent the date during the year that taxpayers choose to get married (0 represents January 1; 1 represents December 31 of a subsequent calendar year); how long to hold a capital asset (0 represents selling immediately after purchase; 1 might represent selling at ten years or some point after25); or the extent to which a taxpayer is motivated by profit to take some activity (0 represents no profit motive; 1 represents complete profit motive). We set aside for now the question of why the tax law seeks to treat these two groups of taxpayers differently, taking this constraint as a given.

Depending on the setting, it may be that one tax regime is more favorable for all taxpayers than the other, or it may be that the relative desirability of the tax regimes differs among taxpayers. To keep things simple, we focus on the case in which taxpayers prefer one tax regime over the other by an identical amount, which we label $\tau$, $\tau = t(1) - t(0) > 0$.

The question we seek to answer is how the tax law should treat individuals with intermediate values of $x$: For any given $x$ between 0 and 1, should the taxpayer be treated like taxpayers with $x = 1$, like taxpayers with $x = 0$, or somewhere in between?

A sharp line is one way the law might assign tax liability to taxpayers with intermediate values of $x$.26 Implementing a sharp line requires selecting the value of $x$ that will serve as the dividing point between classification—we’ll refer to this threshold as $\overline{x}$. The key feature of a sharp line is its all-or-nothing nature: Taxpayers with a value of $x$ on the $x = 1$ side of $\overline{x}$ are treated as if they had $x = 1$; taxpayers with a value of $x$ on the $x = 0$ side of $\overline{x}$ are treated as if they had $x = 0$. Using our notation, tax liability under a sharp line, $t^{SL}$, is given by:

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25 For our purposes, it will not always be necessary to model each possible decision a taxpayer might make as representing a distinct point between 0 and 1—for example, holding periods of fifteen years and sixteen years might both be represented as $x = 1$. In choosing which decisions can be modeled using the same value of $x$, the key assumption is that all such decisions would receive the same tax treatment. Hence, for treatment of capital gains, the endpoint of our scale should be chosen so that it corresponds to a date for which all subsequent dates would be considered long-term.

26 What we refer to as a sharp line, others have referred to as a “bumpy” or “binary” instrument. Alexander, note 7; Kolber, note 7. The unifying theme behind these concepts is the discontinuous relationship between legal inputs (e.g., taxpayer characteristics) and legal outputs (e.g., tax liability).
\[ t^{SL} = t(0) + \tau I(x) \]

where \( I(x) \) is equal to 1 if \( x > \bar{x} \) and equal to 0 if \( x \leq \bar{x} \). As noted above, sharp lines govern many tax law definitions today.

The other approach we consider for differentiating between taxpayers based on their value of \( x \) is a sliding scale. Under a sliding scale, tax liability is determined by first computing liability under the assumption that the \( x = 0 \) regime applies; then under the assumption that the \( x = 1 \) regime applies; and finally, taking the weighted average of liability under the two regimes based on the taxpayer's actual value of \( x \). Using the above notation, we can represent a taxpayer's liability under a sliding scale, \( t^{SS} \), as:

\[ t^{SS} = t(0) + \tau x \]

In contrast with a sharp line, tax liability under a sliding scale varies continuously with the value of \( x \) selected.

To illustrate the two approaches, consider a simple hypothetical world where each person pays $500 per year as a head tax. The legislature then implements a new policy designed to promote homeownership by imposing a $2000 tax on those who rent their housing. Thus \( t(0) \) is $500 and applies to those taxpayers who own a home all year. And \( t(1) \) is $2500 for taxpayers who rent all year. Suppose, however, a taxpayer begins a year renting a home but buys a home later during the same year. How should the law treat taxpayers when they rent for part of the year and own for part of the year? A sharp line might classify taxpayers as "renters" for the year (and increase their tax by the full $2000) if they rent for 183 days or more during the year. In contrast, a sliding scale might impose tax in proportion to the fraction of the year the taxpayer rents.

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27 Note that the sliding scale formula can be expressed as a weighted average of \( t(0) \) and \( t(1) \), with weights given by \( x \): \( t^{SS} = t(1) x + t(0) (1 - x) \).

28 In theory, the distinction between sliding scales and sharp lines itself rests along a continuum, in that the classification variable can be measured at differing levels of granularity. For example, in the homeownership tax example described in the next paragraph, a sliding scale version of the credit might be apportioned based on weeks of homeownership, days, or even hours or minutes. We mostly abstract from such considerations here but note that the optimal degree of granularity for a sliding scale likely differs by setting, potentially balancing trade-offs between efficiency and complexity, similar to the choice between a sliding scale and a sharp line.

29 So \( t^{SL} = 500 + I(Rent) \cdot 2000 \), where \( I(Rent) \) is equal to 1 if the person rents for 183 days or more during the year, and 0 otherwise.

30 Formally \( t^{SS} = t(1) x + t(0) (1 - x) = 2500 \cdot (Fraction Renting) + 500 \cdot (1 - Fraction Renting) \).
purchases a first home on March 15 (the 74th day of the year) and does not sell it during the year would face a tax of:

\[
\left(\frac{74}{365}\right) \times (\$2,000 + \$500) + \left(1 - \frac{74}{365}\right) \times (\$0 + \$500) = \$905
\]

These two possibilities—the 183-day sharp line and the sliding scale—are illustrated in Figure 1.

**Figure 1**

**Sharp Line vs. Sliding Scale Design of Renters Tax**

![Graph](image)

Figure 1 illustrates the relationship between tax liability and days renting under sliding scale and sharp line versions of a hypothetical renters tax.

So far, we have assumed that a sliding scale would transition between the two tax regimes gradually over all intermediate values of \(x\). In some settings, there will be an advantage to transitioning between the tax regimes over some but not all of the intermediate values. For simplicity, we will refer to both of these approaches as sliding scales, but it is worth noting that the smaller the set of values over which the sliding scale transitions between tax regimes, the more it resembles a sharp line.\(^{31}\)

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\(^{31}\) More generally, a sliding scale is just one example of a continuous function that allocates tax liability based on \(x\). In some settings, other (nonlinear) transition rules between
Although many sharp lines can be converted into sliding scales, some pose theoretical or practical challenges. However, some provisions that initially appear difficult to smooth out turn out to be conducive to a sliding scale design. Consider requirements that taxpayers undertake some (discrete) activity once a particular threshold is crossed. For example, most firms are required to issue information returns on credit card payments to payees whose annual credit card payments exceed $20,000. How might this $20,000 sharp line be converted into a sliding scale? If a payee’s transactions for the year total $19,500, should the firm be required to report some but not all of the covered payments? Or similarly, consider the provision requiring employers with fifty or more employees to provide health insurance or pay a penalty. Would a sliding scale require an employer with forty-nine employees to provide partial health insurance (covering some illnesses but not others)?

When the behavior required by a provision is difficult to smooth out (as in the information reporting and health insurance examples), one way to implement a sliding scale is to smooth out the consequences of failing to take the action specified by the law. That is, rather than phase in the requirement to provide health insurance, a sliding scale version of the employer health insurance mandate might phase in the fee for failing to provide insurance in proportion to the size of the employer’s workforce. The policy would then be recast: Firms with, say, between forty and sixty workers would be required to either provide insurance or pay a penalty—the amount of which would gradually increase with the size of the employer’s workforce. Thus, a forty-nine-employee firm that opted against providing health insurance would incur a penalty, but the magnitude of the penalty would be slightly smaller than that owed by a fifty-one-employee firm making the same decision. In principle, this approach could also work in some settings where the consequence for noncompliance is tax regimes may offer efficiency or other advantages. We limit our focus to sliding scales to highlight the basic trade-offs between sharp lines and other approaches. With sufficient flexibility, any tax function that is continuous in $x$ can approximate a sharp line to an arbitrary degree. For a discussion of continuity and linearity in a closely related setting, and the limits to what a linear (i.e., sliding scale) type approach can achieve, see Jeff Strnad, Taxing New Financial Products: A Conceptual Framework, 46 Stan. L. Rev. 569, 597-600 (1994).

One might instead say that the sliding scale version of the provision contains a penalty for noncompliance, but this language is potentially misleading to the extent it suggests that compliance is an all-or-nothing condition. Under a sliding scale, the tax incentive varies based on the degree to which taxpayers conform their behavior to the objectives of the statute. Hence, if the statute’s objective is for large employers to provide health insurance to their employees, the statute’s goals are more frustrated—in this sense—when a fifty-one-employee firm fails to provide coverage than when a forty-nine-employee firm fails to do so.
nonmonetary, so long as it is scalable. In criminal tax cases, for example, one could scale a convicted taxpayer's prison sentence based on the degree to which the law was violated (or other factors, such as the taxpayer's culpability). On the other hand, a sliding scale is harder to conceive of when the penalty for failing to satisfy the requirement takes the form of a social sanction.

Other parts of the tax law are less susceptible to a sliding scale design. Broadly speaking, the challenges in implementing a sliding scale arise either because the outcome tied to a classification does not lend itself to being smoothed out or because the classification variable itself is inherently binary.

First, consider settings where the outcome of a classification test does not lend itself to being smoothed out. This issue arises infrequently in tax, because most aspects of the tax law ultimately affect tax liability, and tax liability is continuous and can be arbitrarily scaled. In some cases, however, other tax-related outcomes do hinge on classifications, and those can be more difficult to smooth. Certain procedural requirements fall into this category. For example, the tax court provides taxpayers with the option of arguing their tax deficiency case under a simplified procedure if the total unpaid tax is less than $50,000. It is difficult to imagine smoothing out this line: Would a taxpayer alleged to owe $51,000 try the case twice, once under the simplified rules and once under standard rules, and pay some average of the judgments? Eligibility for tax assistance programs like the Volunteer Income Tax Assistance program and Low-Income Taxpayer Clinics raise similar considerations.

To the extent that convictions for criminal tax violations matter in themselves, apart from the corresponding prison sentence or fine (which are continuous and therefore can be smoothed out), they too illustrate a tax-related outcome that is difficult to assign via a sliding scale.

33 Depending on the setting, a sliding scale could be implemented in criminal tax cases by imposing a (relatively small) criminal penalty for levels of activity that are below the threshold that would trigger a criminal conviction under current law, or the criminal penalty could start phasing in at the current threshold for conviction. Of course, such factors may already be considered in sentencing decisions. See Adam J. Kolber, The Bumpiness of Criminal Law, 67 Ala. L. Rev. 855, 869-70 (2016). Note that this approach runs into difficulty if a large part of the motivational force comes not from the specific sentence, but from the discrete fact of violating the law or of conviction.


35 One probably undesirable approach for converting a sharp line to a sliding scale in such cases would be to randomize outcomes, with the probability varying continuously in $x$. This approach would approximate the efficiency properties of the sliding scale (at least in expectation) but the complexity and horizontal equity properties of the sharp line. See our discussion of uncertainty in Part II.F.
In other cases, a sliding scale will be a poor fit because the variable underlying a classification test is itself inherently binary. Consider a corporate residency determination, for example; a corporation can either charter in the United States or abroad. If a classification depends solely on this factor, it is difficult to imagine how it could be smoothed out, since the classification variable itself can take on values of only 0 or 1. Similarly, if a classification turns solely on whether a taxpayer knowingly violated the law, implementing a sliding scale would require differentiating between various degrees of “knowingly.”

In practice, several factors limit the importance of this consideration. First, although classifications that turn exclusively on a single binary variable cannot be smoothed out, sliding scales can still be feasible when multiple binary variables enter into a multifactor test—a possibility we discuss in Part VII.F. Second, classifications that turn on a binary classification variable are often motivated by goals that are themselves nonbinary and amenable to a sliding scale. For example, consider a classification that turns on whether a taxpayer is married. Although legal marital status is binary, such provisions might actually be motivated by considerations for which marital status serves as a convenient proxy, such as financial interdependence or household size. In such cases, one way to smooth out the classification is for the legal test to incorporate one or more of these underlying factors rather than focusing exclusively on formal marital status alone. Similarly, a criminal tax provision that turns on whether a violation of the law was knowing might be replaced by a sliding scale focusing on factors related to culpability more generally.

In the Parts that follow, we consider a number of factors that shape the relative merits between sharp lines and sliding scales.

II. Efficiency Considerations

This Part compares the efficiency properties of sharp lines and sliding scales. In the cases we consider, we find that sliding scales tend to be more efficient than sharp lines. However, we also identify situations when sharp lines are in fact more efficient. This occurs when—absent taxes—few taxpayers would locate near the sharp line. The magnitude of the difference in efficiency between sharp lines and sliding scales depends in part on the difficulty or ease with which taxpayers can adjust their behavior on account of the tax and on how directly taxpayers can control the factors that determine their tax liability.
A. Model of Taxpayer Behavior

To compare the efficiency of sharp lines and sliding scales, consider a classification variable x over which taxpayers have at least some control. The level of control that taxpayers exert over x may be complete, such as the date in the year the taxpayer gets married, or imperfect, such as the date in the year that a child is born. If x is entirely outside of the taxpayer’s control, efficiency considerations can safely be set aside (at least with respect to distorting the taxpayer’s behavior relating to x). For simplicity, we will initially focus on the case in which taxpayers have complete control over their value of x.\footnote{36}

To model how taxpayers select x, we impose two assumptions about taxpayer behavior. First, we assume that absent tax considerations, taxpayers have some ideal value of x they would like to select, which we label $x_i$ (the subscript i denotes the taxpayer). All else being equal, taxpayers prefer to select values of x that are as close as possible to their ideal point.\footnote{37}

Second, we assume the incremental cost of locating farther from one’s ideal point grows larger as the distance from the ideal point increases. “Convex adjustment costs” of this form are commonly imposed in economic analyses.\footnote{38} To illustrate, consider taxpayers seeking to reduce the number of days they spend in a high-tax state.\footnote{39} Taxpayers might look at their calendars to identify all the days they had planned to spend in the state; some plans that would take them into the state might be easy to cancel or move to a different location. These would be the first to go. Further reductions would require costlier adjustments to their behavior—perhaps a meeting could be rescheduled but only at substantial inconvenience. And some adjustments—such as switching jobs to work for an employer in a different state or ceasing to visit a family member who lives in the high tax jurisdiction—would be so costly that the tax consequences would be unlikely to justify them.\footnote{40}

\footnote{36} We explore the case in which taxpayers imperfectly control x in Part II.G.
\footnote{37} The magnitude of the cost to locating away from one’s ideal point may vary based on the time horizon being considered. Short-term adjustment costs may exceed long-term ones, for example.
\footnote{38} See, e.g., Shane Singh, Linear and Quadratic Utility Loss Functions in Voting Behavior Research, 26 J. of Theoretical Pol. 35 (2014) (discussing this issue in one setting).
\footnote{39} For a vivid illustration of this point, see James B. Stewart, Tax Me If You Can, The New Yorker, Mar. 12, 2012 (discussing taxpayers seeking to avoid New York City residence).
\footnote{40} As a different example, consider a taxpayer holding a capital asset that she would prefer to sell but continues to hold in order to obtain long-term capital gains treatment. If the taxpayer would, absent taxes, sell stock A and buy stock B because she believes B will outperform A, the gap in future expected profits would compound with each additional day.
Absent tax considerations, individuals would locate at their ideal point, \( x^*_i \). After taxes are introduced, taxpayers balance the desire to locate near their ideal points with the desire to minimize tax liability. As we will see, the specific values of \( x \) that taxpayers choose, as well as the tax's deadweight loss, will depend on which form of rule the law adopts.

**B. Behavioral Effects of a Sharp Line**

We first consider the behavioral effects of imposing a sharp line rule, with cutoff point \( \bar{x} \).\(^{41}\) To illustrate how this type of rule affects behavior, we will examine how four different taxpayers would respond. The taxpayers are identical to one another except for their ideal points, as indicated in Figure 2.

Consider first taxpayer \( A \), whose ideal point is denoted by \( x_A^* \). Because \( x_A < \bar{x} \), choosing her ideal point results in \( A \) being taxed under the favorable tax regime and yields tax liability equal to \( t(0) \). Because there is no way for \( A \) to reduce her liability by adjusting her choice of \( x \), she will select her ideal point, \( x_A = x_A^* \). The same logic holds for other taxpayers with ideal points between 0 and \( \bar{x} \).

Next consider taxpayer \( B \), whose ideal point is just to the right of \( \bar{x} \). Absent tax considerations, \( B \) would also select his ideal point. But because a small reduction in \( x \) yields substantial tax advantages, \( B \) will find it worthwhile to choose \( x_B = \bar{x} \) instead of \( x_B^* \). Intuitively, \( B \) is worse off for choosing to locate away from his ideal point, but incurring this cost is worth it for him because locating at the cutoff allows \( B \) to remain close to his ideal point while also allowing him to significantly reduce his tax liability. Note that \( B \) would never choose \( x_B < \bar{x} \) because he can achieve all of the tax benefits by moving to \( \bar{x} \), and any point to the left of \( \bar{x} \) would be farther from his ideal point.
Next consider taxpayer C, who, like B, has an ideal point to the right of \( \bar{x} \), but whose ideal point is farther from \( \bar{x} \). For C to change her behavior because of the tax, it must be that her ideal point is close enough to the cutoff that switching to \( \bar{x} \) to achieve the tax savings is worth the sacrifice in terms of selecting a value of \( x_C \) away from \( x_C^* \). Suppose that C does find it worthwhile to change her behavior in this way. Like B, C will choose to locate exactly at \( \bar{x} \), since moving farther to the left would cause her to be farther from her ideal point and would yield no additional tax savings. Note that, all else being equal, C is worse off than B because although both are taxed under the preferable regime, C is farther from her ideal point.

Finally, consider taxpayer D, whose ideal point is to the right of both \( x_B^* \) and \( x_C^* \). Like B and C, tax considerations would motivate D to select \( x_D = \bar{x} \), but the utility cost of doing so would be greater than for B or C, since \( x_D^* \) is even farther from \( \bar{x} \). Assuming that the utility cost from moving to \( \bar{x} \) exceeds the tax reduction that D would achieve from doing so, D will remain at \( x_D^* \). Note that there is no benefit to D from moving any closer to \( \bar{x} \) while still remaining to the right of it, since doing so would not reduce his tax liability.

To summarize the effect of the sharp line on taxpayers' behavior, we can divide taxpayers into three categories, based on their ideal points.
Someone whose ideal point is to the left of \( \bar{x} \) will choose \( x_i = x_i^* \). Someone whose ideal point is slightly to the right of \( \bar{x} \) will choose \( x_i = \bar{x} \). And someone whose ideal point is sufficiently to the right of \( \bar{x} \) will choose \( x_i = x_i^* \). Thus, under a sharp line, some taxpayers adjust their choice of \( x \) because of the tax, while others do not. Those who adjust their behavior select \( \bar{x} \). Those who do not adjust their behavior select their ideal points.

### C. Efficiency Properties of a Sharp Line

What are the efficiency costs of the sharp line? After adjusting their behavior in response to the tax, A, B, and C each pay a tax of \( t(0) \), and D pays a tax of \( t(0) + \tau \). The deadweight loss of a tax is equal to the amount by which it reduces taxpayer welfare over and above the revenue it collects. Here, taxpayers are worse off by the amount of tax they pay. In addition, those taxpayers who adjust their choice of \( x \) because of the tax are worse off because they are no longer at their ideal point. In Figure 2, therefore, taxpayers B and C are the ones who generate deadweight loss. Taxpayer B generates only a little deadweight loss because \( \bar{x} \) (where B ends up) is close to \( x_B^* \). Taxpayer C generates more deadweight loss than B, because the value of \( x \) she ends up choosing (\( \bar{x} \)) is farther from her ideal point. Taxpayers A and D generate no deadweight loss because they are worse off only from the tax revenue they transfer to the government. Thus, the more taxpayers there are near B, and especially the more taxpayers there are near C, the larger the deadweight loss generated by the sharp line.\(^{42}\)

We illustrate the efficiency consequences of a sharp line in Figure 3, which simulates the deadweight loss (as a percentage of revenue collected) generated as a function of a taxpayer’s ideal point.\(^{43}\) In our analyses, we assume that the sharp line threshold is set to \( \bar{x} = 0.5 \).\(^{44}\) As suggested by our discussion above, the simulation results reflect the

\(^{42}\) Note that the magnitude of the deadweight loss is shaped in part by the costs to taxpayers of deviating from their ideal points, since such costs determine how many taxpayers behave like C versus D in the prior example. In the extreme case, where the costs of deviating from one’s ideal point are so great that taxpayers never choose to adjust their behavior in response to the tax, there is no deadweight loss.

\(^{43}\) The simulation assumes that taxpayers face quadratic loss from adjusting their behavior away from their ideal point and linear disutility from paying tax: \( U_i = -a(x_i - x_i^*)^2 - t(x_i) \). The analysis reported in Figure 3 assumes \( a = 3 \), but we obtain qualitatively similar results for \( a \in \{0.5, 1, 5, 10\} \). The deadweight loss generated by taxpayers with a given value of \( x_i^* \) corresponds to the additional revenue the government could raise from such taxpayers under a lump-sum tax without making the taxpayer worse off than under the distortive tax.

\(^{44}\) A more fulsome analysis of this issue would need to account for the underlying rationale for differentially classifying taxpayers by \( x \); this is because setting \( \bar{x} = 1 \) would eliminate all deadweight loss arising from the classification, at the cost of eliminating the classification in the first place.
fact that no deadweight loss is generated by taxpayers with ideal points below the line or taxpayers with ideal points sufficiently above the line that their behavior is not affected by the tax. In contrast, the sharp line induces taxpayers with ideal points slightly to the right of the line (between 0.5 and 0.65 in our simulation) to select $x = \bar{x}$. Within this range, the farther from the line a taxpayer's ideal point is, the more deadweight loss the taxpayer's behavior generates.

**Figure 3**

**DEADWEIGHT LOSS BY TAXPAYER TYPE UNDER A SHARP LINE**

Figure 3 shows the contribution to deadweight loss of taxpayers with varying ideal values of the classification variable under a sharp line. The results are derived from the simulation exercise described in the text.

**D. Behavioral Effects of a Sliding Scale**

Now consider the behavioral effects of a sliding scale, as illustrated in Figure 4. As in Figure 2, Taxpayers A, B, C, and D are drawn based on the location of their ideal points. First, consider taxpayer A. Under the sharp line, A faced no tax incentive to choose a value of $x$ other than her ideal point. Under the sliding scale, if A reduces $x_A$ below $x_A^*$, she will reduce her tax liability as well. However, if she chooses too low a value of $x$, the disutility from locating away from her ideal point will exceed the benefit of reducing her tax liability. Hence, she will choose
some value of $x$ that is less than $x_A^*$, but she would generally not reduce her choice of $x$ all the way to 0.$^{45}$

**Figure 4**

**Behavioral Effects of a Sliding Scale**

![Figure 4](image_url)

Figure 4 illustrates the incentive effects generated by a sliding scale for taxpayers with varying ideal values of the classification variable.

The analysis is the same for taxpayers $B$, $C$, and $D$. As long as the cost of diverging from one’s ideal point is the same for all taxpayers, the tax will cause all taxpayers to reduce their $x$ in a similar manner.$^{46}$ For a taxpayer like $B$ (i.e., one whose ideal point is sufficiently close to the line), the reduction in $x$ induced by the sliding scale will be similar to or can even exceed the reduction in $x$ induced by the sharp line; this is because such a taxpayer would adjust her behavior by only a very small amount in response to the sharp line but would adjust her behavior in the same manner as everyone else in response to the sliding scale. In contrast, taxpayers like $C$ would select a value of $x$ farther

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$^{45}$ This follows from the assumption that the marginal disutility of locating farther away from one’s ideal point increases in the distance from the ideal point. Someone with an ideal point close enough to 0 would select $x = 0$, simply because $x$ cannot be reduced below that amount.

$^{46}$ The precise response depends on taxpayers’ utility functions. Under the utility function assumed in the simulations, for example, taxpayers reduce $x$ by a uniform amount.
from their ideal point under the sharp line than under the sliding scale because the potential tax savings from moving to \( \bar{x} \), and enjoying the full savings of the low-tax regime, are more than the potential savings from moving to a lower value of \( x \) under the sliding scale.

**E. Efficiency Properties of the Sliding Scale**

Figure 5 illustrates the deadweight loss under a sliding scale using the simulation that generated Figure 3. Because all the taxpayers \( A, B, C, \) and \( D \) adjust their behavior by the same amount, they each generate the same deadweight loss. As with the sharp line, the magnitude of the deadweight loss associated with the sliding scale depends on the cost to taxpayers of deviating from their ideal points, relative to the tax benefits. When doing so is prohibitively expensive, sliding scales (like sharp lines) do not generate much deadweight loss.

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47 Taxpayers with ideal points sufficiently close to 0 will simply move to 0. Consequently, these taxpayers tend to generate less deadweight loss than others who are not limited in the amount they can adjust their behavior in response to the tax. This is not reflected in Figure 4 because our smallest displayed point (\( x^*_i = 0.01 \)) is not small enough for such taxpayers to want to move all the way to 0 under the utility function we assume.
Figure 5 shows the contribution to deadweight loss of taxpayers with varying ideal values of the classification variable under a sliding scale. The results are derived from the simulation exercise described in the text.

F. Comparing the Efficiency of a Sharp Line and Sliding Scale

Assessing the relative efficiency of a sharp line and sliding scale requires determining whether the deadweight loss from the change in behavior of taxpayers like B and C under the sharp line exceeds the deadweight loss from the change in behavior of all taxpayers under the sliding scale. On the one hand, more taxpayers change their behavior under the sliding scale, and hence generate at least some deadweight loss. On the other hand, because the sharp line induces larger changes in behavior for those taxpayers who do respond to it, the overall magnitude of the deadweight loss can be larger in that case.

The question is therefore whether there are relatively more taxpayers like C (who favor the sliding scale) or relatively more taxpayers like A and D (who favor the sharp line). The presence of taxpayers like B—who adjust their behavior modestly in response to

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48 In comparing the deadweight loss generated under the two policies, we hold fixed the amount of revenue raised by the tax.
both the sharp line and the sliding scale—has an approximately neutral effect, since both of these changes are of roughly equal magnitude.49

To illustrate this point in more detail, we draw on a simulation exercise50 to explore the following three possibilities for how taxpayers' ideal points are distributed between 0 and 1:

1. $x_i^*$ is evenly distributed across each value between 0 and 1;
2. $x_i^*$ is distributed according to a bell curve, with a peak at $\bar{x}$; or
3. $x_i^*$ has a bimodal distribution, with most ideal points concentrated near 0 and 1.

**Figure 6
Potential Distributions of Taxpayer Ideal Points**

Our first result is that when ideal types are evenly distributed across each value of $x$, a sliding scale tends to be more efficient than a sharp line.51 The intuition for this result is that although the sliding scale causes more people to change their behavior than the sharp line, the magnitude of the adjustments under the former is smaller than the magnitude of the adjustments under the latter. Because of the

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49 The analysis thus far assumes that taxpayers fully consider both sharp lines and sliding scales when determining their behavior. Depending on the magnitude of the taxes, taxpayers may not account for the two designs equally, which can affect the way they shape taxpayer behavior. We discuss such issues in Part VI.

50 As with the simulations reported in Figures 3 and 5, we assume taxpayer utility is quasi-linear in taxes and quadratic in loss from distance to one's ideal point, $U_i = -\alpha(x_i - x_i^*)^2 - t(x_i)$. The simulations adopt the following procedure. First, we select a tax rate $\tau_{SS}$ and compute the amount of tax revenue generated by that $\tau_{SS}$ under the sliding scale ($R$), where $R = \int t_{SS} x_i$. Second, we compute the tax amount $\tau_{SL}$ that generates $R$ under the sharp line. Third, we compare the deadweight loss generated by the sliding scale with rate $\tau_{SS}$ to the sharp line with tax amount $\tau_{SL}$. Throughout, we assume the sharp line is placed so that $\bar{x} = 0.5$. The simulations presented in the body of the paper assume $\tau_{SS} = 0.05$ and $\alpha = 3$, but we obtain qualitatively similar results for $\tau \in \{0.005, 0.01, 0.03, 0.05, 0.07, 0.1\}$ and $\alpha \in \{0.5, 1, 3, 5, 10\}$.

51 More precisely, our claim is that a sliding scale is more efficient than a sharp line for the case in which taxpayers' ideal points are uniformly distributed and the loss from deviating from one's ideal point is quadratic. A proof of this result is contained in the Appendix.
convexity of adjustment costs—that is, the idea that incremental deviations away from one’s ideal point generate more disutility the farther away from the ideal point one starts out—those taxpayers who change their behavior by a lot in response to the sharp line generate much more deadweight loss than those taxpayers who adjust their behavior by a little in response to the sliding scale. Using the language of our previous example, although there are more A’s and D’s than C’s when there is a uniform distribution of ideal points, this ratio—of \((A+D)\) to \(C\)—is not large enough to account for how much extra deadweight loss each \(C\) adds.\(^2\) Our simulation analysis illustrates this result in the first panel of Figure 7.

Likewise, if taxpayers’ ideal points follow a normal distribution around the threshold \((\bar{x})\), the efficiency advantage of the sliding scale is even greater. Intuitively, the normal distribution guarantees that more taxpayers are like \(C\), whose ideal points are just close enough to the threshold to be willing to adjust their choice of \(x\) by a substantial amount in response to the tax.

**Figure 7**

**DEADWEIGHT LOSS BY TAXPAYER IDEAL POINT DISTRIBUTION**

![Figure 7](image)

Figure 7 compares deadweight loss for a sharp line and sliding scale under varying assumptions about the distribution of taxpayer ideal values of the classification variable. The results are displayed as a percentage of the deadweight loss generated by the sharp line and derived from the simulation exercise described in the text.

Last, if the \(x_i^\ast\)’s have a bimodal distribution, and are concentrated around 0 or 1, the sharp line can be more efficient. The reason why is that this distribution ensures that not many taxpayers have ideal points near the threshold, reducing the amount of deadweight loss the sharp line generates. In the language of our earlier illustration, the bimodal distribution guarantees that there are relatively few taxpayers in categories \(B\) or \(C\), which limits the deadweight loss generated by the sharp line. More generally, this example highlights that if there is a “dip” in the distribution of taxpayer ideal points, such that the line can

\(^2\) In contrast, if the disutility from deviating from one’s ideal point was linear, rather than convex, the relative efficiency of the two policy types would depend simply on which induced a larger total change in taxpayer behavior, regardless of whether individual taxpayers changed their behavior by a lot or a little.
be drawn in a way that will not distort most taxpayers’ behavior, a sharp line can be more efficient than a sliding scale.

Given that the efficient choice between sharp lines and sliding scales depends on the distribution of taxpayers’ ideal types, a reasonable question to ask is how might a policymaker learn about the distribution of ideal types in any practical application? The key question is whether the distribution contains a sufficiently large “dip” so that a sharp line could be drawn in a relatively non-distorting manner.

In cases in which there is no relevant existing tax, policymakers can examine the distribution of ideal points directly. For example, the Tax Reform Act of 1986 (TRA86) taxed long-term capital gains at the same rate as ordinary income, eliminating the incentive to hold assets for just over a year to obtain a more favorable rate. Thus, data on how long taxpayers held capital assets in the wake of TRA86 would reveal taxpayers’ ideal distribution of holding periods ($x^*$) during that period.

If the tax in question already exists, estimating $x^*$ is more difficult, but usually feasible (at least with sufficiently strong assumptions). One option is to look at the behavior of groups exempted from the tax, if there are any. Often, however, policymakers will need to look to data on actual choices made subject to the tax and attempt to back out the distribution of $x^*$. As described above, if the tax is a sharp line, its effects on behavior take the form of inducing a dip in the distribution just on the tax-disfavored side of the line. If the observed distribution does not have any other dips, therefore, this implies that $x^*$ likely does not contain a dip.

To illustrate, consider Figure 8, which shows the distribution of taxpayers’ holding periods of capital assets (with gains). The solid line is the actual holding period data. As predicted by the theory, the figure shows a spike in sales just after the one-year holding period threshold needed to obtain the lower capital gains rate, and disproportionately few sales just prior to the one-year mark. The lack of any other substantial dip in the distribution suggests that the distribution of

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53 See Roberton Williams, Tax Rates on Capital Gains, Tax Notes, Jan. 9, 2012.
54 For example, with data on actual capital asset holding periods, one could look at exempt entities like universities, charities, pension funds, or assets in 401(k) funds. The difficulty with this method is extrapolating from the tax-exempt group to the whole population.
55 When the observable distribution of $x$ comes from decisions made under a sliding scale, backing out the underlying distribution of $x^*$ typically requires more assumptions. If we assume, as the model does in Part I, that all taxpayers respond to the tax in a uniform fashion, then we can conclude from the absence of dips in the observable distribution of $x$ that $x^*$ lacks a dip as well (or vice versa if the observed $x$’s have a dip, so do the $x^*$’s).
taxpayers' ideal holding periods ($x^*$), which we approximate with a dashed line in Figure 8, is relatively smooth.\footnote{To be clear, our estimate of $x^*$ here is an approximate one based only on visual inspection of the data. A more sophisticated analysis would model $x^*$ by formally estimating the size of the discontinuity at the one-year threshold and then assigning the “excess” sales that occur just after the one-year threshold to an $x^*$ earlier in time.}

**Figure 8**

**Total Capital Gains Recognized by Holding Period Length**

![Graph showing total capital gains recognized by holding period length.](image)

Note: The horizontal axis is the number of weeks before and after the one-year holding period. Source: Dowd & McClelland, note 53 (using data from the Statistics of Income from 2016).

Finally, in addition to knowing whether a sharp line or sliding scale is more efficient in a particular context, it is also important to know how much more efficient one approach is than the other, especially in cases where the more efficient approach is less desirable on other grounds, such as complexity. As indicated in the above discussions, the deadweight loss generated by both a sharp line and sliding scale depends not only on how much taxpayers adjust their behavior in response to the tax, but also on how costly it is for them to do so. Thus, in settings in which it is relatively costless for taxpayers to choose values of $x$ that are far from their ideal points, the deadweight loss generated by either the sharp line or the sliding scale will tend to be small. In contrast, when it is costly for taxpayers to adjust their behavior, but the incentives generated by the tax instrument are strong enough that taxpayers still do so, the amount of deadweight loss will be
larger, and hence, the efficiency differences between sliding scales and sharp lines will be more important as well.

G. Uncertainty

Thus far, we have assumed that a taxpayer’s liability is a deterministic (i.e., nonrandom) function of her choice of $x$. In some settings, a taxpayer’s liability might remain uncertain even after she selects $x$. This might occur, for example, if the tax authority observes $x$ only imperfectly, as when tax liability turns on whether a taxpayer’s primary purpose in incurring an expense was business or personal. In other cases, uncertainty might arise because the applicable law takes the form of a standard rather than a rule, in which tax liability is determined by applying some general, non-dispositive principle to the situation at hand, such as whether an employer-provided meal is incurred “for the convenience of the employer” or whether a particular transaction will be characterized as a “sham.” In such cases, taxpayers may be uncertain as to the tax liability they will face based on their choice of $x$.

To apply our model to such settings, suppose the tax authority observes the value of the classification variable that the taxpayer selects with error. Using the notation of the model, the taxpayer continues to select $x$, but tax liability under the sharp line and sliding scale each depend on $\bar{x}$, where $\bar{x} = x + \epsilon$, and where $\epsilon$ is a normally distributed error term with mean zero:

$$\text{Sharp Line: } t^{SL} = t(0) + \tau I(\bar{x})$$
$$\text{Sliding Scale: } t^{SS} = t(0) + \tau \bar{x}$$

Altering the model in this way reveals that uncertainty tends to reduce the magnitude of the efficiency differences between sliding scales and sharp lines. The reason why is that the uncertainty between a taxpayer’s choice of $x$ and ultimate tax liability effectively smooths out the taxpayer’s expected tax liability as a function of $x$, making the taxpayer’s incentives under the sharp line resemble those faced under a sliding scale. With uncertainty, a taxpayer who selects a value of $x$ just below the line may still be treated by the tax authority as having

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58 A different form of uncertainty, which we do not consider here, is when taxpayers have imperfect control over $x$—for example, when $x$ is uncertain given the taxpayer’s behavior. This form of uncertainty enters the model differently because it creates uncertainty in taxpayers’ direct utility from $x$ in addition to their tax liability.

59 Although uncertainty makes the instruments more similar in expectation, risk-averse taxpayers may prefer sliding scales in the presence of uncertainty because sliding scales reduce the variance of realized tax liability relative to sharp lines.
selected a value of \( x \) that is above the line. Conversely, a taxpayer who selects a value of \( x \) just above the line may end up being treated by the tax authority as having selected a value of \( x \) that is below the line. Thus, uncertainty reduces taxpayers' incentives to make drastic changes to their behavior to locate on the tax-minimizing side of a sharp line.\(^{60}\)

Figure 9 illustrates expected tax liability as a function of \( x \) under both a sharp line and a sliding scale in the presence of uncertainty. The greater the uncertainty (i.e., the larger the standard deviation in the distribution of \( \epsilon \)), the more the expected tax liability schedule for the sharp line resembles that of the sliding scale, and hence, the smaller the efficiency differences between the policies will be. Thus, in Figure 9 in the left-hand panel, when the standard deviation of \( \epsilon \) is small, the sharp line regime continues to look roughly similar to the situation without uncertainty, but with some smoothing out of expected liability around the threshold. By contrast, when the degree of uncertainty is large, the sharp line regime more closely resembles the sliding scale (Figure 9 right-hand panel).\(^{61}\)

The simulation analysis presented in Figure 10 compares sharp lines and sliding scales in the case in which \( x_i \) is uniformly distributed, first without uncertainty and then in the presence of different levels of uncertainty.\(^{62}\) As illustrated in the figure, the sliding scale continues to generate less deadweight loss relative to the sharp line in the presence of uncertainty, but the efficiency properties of the two instruments are more similar than when no uncertainty is present.

Thus, whether for good or bad, the efficiency differences between sharp lines and sliding scales are less pronounced when there is more uncertainty between taxpayers' decisions and their ultimate tax

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\(^{60}\) This point is related to the classic result that legal uncertainty has an ambiguous effect on a deterrence policy's behavioral effects. John E. Calfee & Richard Craswell, Some Effects of Uncertainty on Compliance with Legal Standards, 70 Va. L. Rev. 965, 965-67 (1984); John E. Calfee & Richard Craswell, Deterrence and Uncertain Legal Standards, 2 J.L. Econ. & Org. 279, 279-80 (1986). The similarity is due to the fact that uncertainty converts a sharp line into an instrument with similar incentive properties as a sliding scale (at least in expectation). See Emily Cauble, Safe Harbors in Tax Law, 47 Conn. L. Rev. 1385, 1420 (2015), (comparing the distortive effects of a safe harbor—that is, a sharp line rule—with an uncertain standard, that is, an instrument that generates similar incentives to a sliding scale); Alex Raskolnikov, Probabilistic Compliance, 34 Yale J. on Reg. 491 (2017) (studying how uncertainty affects decisions by the taxpayer about efforts to ensure compliance with a legal standard). Some of these points also arise in David Weisbach's comparison of (sharp line) rules and standards. See David Weisbach, Formalism in the Tax Law, 66 U. Chi. L. Rev. 860, 872-75 (1999).

\(^{61}\) The sliding scale in Figure 9 is no longer exactly a straight line because around 0 errors can only push up \( x \), resulting in slightly higher expected tax liability for taxpayers choosing \( x = 0 \) than in the case without uncertainty. The reverse is true around 1: Errors can only push down \( x \).

\(^{62}\) The simulation imposes the same assumptions about taxpayer utility functions as in the simulations described above.
liability. In the remainder of the Article, we consider how the choice between sharp lines and sliding scales affects tax policy considerations apart from efficiency.

**FIGURE 9**

**EXPECTED TAX LIABILITY UNDER A SHARP LINE AND SLIDING SCALE WITH UNCERTAINTY**

Figure 9 shows expected tax liability as a function of the classification variable under a sliding scale and a sharp line. The left panel assumes a standard deviation of the noise term of 0.05; the right panel assumes a standard deviation of the noise term of 0.2.

**FIGURE 10**

**RELATIVE DEADWEIGHT LOSS BY LEVEL OF UNCERTAINTY**

Figure 10 shows the deadweight loss generated by a sliding scale relative to the deadweight loss generated by a sharp line at varying levels of uncertainty. The results are derived from a
simulation exercise, described in the text. The left column assumes no uncertainty. The second-to-left column assumes a standard deviation of 0.05 for the noise term. The second-to-right column assumes a standard deviation of 0.2 for the noise term. The right column shows deadweight loss generated from a sharp line as a reference. Note that the right column is mechanically equal to 100, regardless of the level of uncertainty.

III. COMPLEXITY CONSIDERATIONS

In addition to differing in the way they affect taxpayer behavior, sliding scales and sharp lines can differ greatly in their complexity. A particular test for classifying taxpayers may be complex in some dimensions and simple in others. In this Part, we consider both the computational and informational complexity of sliding scales and sharp lines, and the implications of each for the complexity of tax filing and tax planning.

Our main conclusion is that sliding scales can complicate the tax filing process relative to sharp lines, but the difference in complexity between the instrument types differs greatly by setting. Our conclusion is similarly nuanced with respect to tax planning: Sliding scales are more complex in some dimensions and less complex in others. Below, we outline the main considerations that determine which approach is more complex in a particular context.

A. Computational Complexity

Computational complexity refers to the complexity of determining an individual's tax liability given all of the relevant information about the individual (e.g., income, family size, etc.). It is easy to see that sliding scales are more computationally complex than sharp lines. Under a sharp line, taxpayers determine their liability by first determining which tax regime applies, and second, calculating their liability under that regime. Implementing the first step simply requires assessing whether the classification variable \( x \) is above or below the threshold \( Y \). Under a sliding scale, in contrast, taxpayers must compute their tax liability under both regimes, and then take a weighted average of the two resulting liabilities based on \( x \). For most people, calculating a weighted average based on \( x \) will be more difficult than comparing \( x \)

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64 For an extended discussion of computational versus informational complexity, see Jacob Goldin, Tax Benefit Complexity and Take-Up: Lessons from the Earned Income Tax Credit, 72 Tax L. Rev. 59 (2018).
to $x$, and it is obviously more work to calculate one's liability under two different tax regimes than under one.

To illustrate, consider the choice of rules for whether taxpayers can claim a child on their return, based on the child's birth date. Under a sharp line, taxpayers simply determine whether the child's birth date falls before or after the cutoff, and then calculate liability once, either claiming or not claiming the child based on when the child was born. Under a sliding scale, taxpayers would calculate their liability twice, once assuming they claim the child and once assuming they do not, and then take a weighted average of the two results based on the date the child was born. The sliding scale calculation is more complicated, since it requires calculating liability twice, and entails the additional step of taking the weighted average.

In some cases, the computational complexity of a sliding scale is more extreme. For example, the choice of applicable tax regime may have implications not only for determining the tax consequences of a particular transaction, but also of subsequent transactions or events. One example is the 80% control requirement for § 351 exchanges in corporate taxation. Smoothing out the test for a nonrecognition provision of this form has implications not only for the transaction itself (i.e., how much gain or loss is recognized), but also for subsequent transactions that depend on the corporation's basis of the contributed property or the taxpayer's basis in the received stock.

The additional computational complexity from smoothing out a sharp line may be even more extreme than in the prior example in settings where the outcome of the test determines which of two very different tax regimes applies. To illustrate, consider the hundred-

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65 In some cases, sliding scales can be designed to avoid requiring taxpayers to compute their entire tax liability twice, such as when the outcome of the sliding scale is the share of an expense the taxpayer is permitted to deduct.

At the other extreme, with multiple sliding scale rules in place, the number of calculations a taxpayer must perform can increase exponentially. For example, suppose that both marital status ($M$) and state residency ($R$) enter into a sliding scale classification based on when in the year those events occurred. In this case, the taxpayer must calculate tax liability under four scenarios: married resident ($M = 1, R = 1$), married nonresident ($M = 1, R = 0$), single resident ($M = 0, R = 1$), and single nonresident ($M = 0, R = 0$). Tax liability is a function of both $M$, and $R$, $t(M, R)$. Tax liability under a sliding scale is then given by:

$$t^{ss} = t(0,0) + x_M [t(1,0) - t(0,0)] + x_R [t(0,1) - t(0,0)] + x_{MR} [t(1,1) - t(1,0) - t(0,1) + t(0,0)]$$

where $x_M$, $x_R$, and $x_{MR}$ respectively denote the share of days during the year that the taxpayer was married, living in the state, and married while living in the state. Note that when the tax consequences of marriage and state residency are independent, the last term of this equation is equal to 0. Equivalently, we can express this formula as a weighted average, based on the share of the year the taxpayer falls into each of the four marital status by residency categories:

$$t^{ss} = (1 - x_M - x_R + x_{MR})t(0,0) + t(1,0)(x_M - x_{MR}) + t(0,1)(x_R - x_{MR}) + t(1,1)x_{MR}$$
shareholder limitation for S corporations. In theory, one might smooth this requirement out as follows: Over some range of shareholders, say from 90 to 110, the entity would be taxed partly under subchapter C and partly under subchapter S. For example, if the entity had 95 shareholders, it could be taxed 75% as an S corporation and 25% as a C corporation. How might this be implemented? One option would be to pretend there were in fact two corporations, an S corporation that earned 75% of the income (and incurred 75% of the expenses, etc.), and a C corporation that earned 25% of the income. This approach would smooth out the shareholder requirement, but would generate substantial computational complexity for taxpayers—who would have to compute taxes under two entirely different regimes.

A different example that illustrates a similar challenge relates to the qualifications for tax-exempt entity status, such as the requirement that a § 501(c)(4) entity not be engaged primarily in political activity. In theory, these tests can be smoothed out, but the computational complexity that doing so would cause is likely not worth the effort.

B. Informational Complexity

Informational complexity refers to the difficulty in obtaining the information upon which the determination of one’s tax liability depends. As with computational complexity, a sliding scale tends to be more informationally complex than a sharp line. There are two reasons for this. First, a sliding scale requires more granular information on the classification variable than does a sharp line. To determine the applicable tax regime under a sharp line, the taxpayer must know only whether \( x \) exceeds the threshold \( \bar{x} \). Under a sliding scale, the taxpayer must know the precise value of \( x \), since that value determines the relative weighting between the two tax regimes.

The second reason a sliding scale tends to be more informationally complex is that determining liability under the second potential tax regime—the one that would not apply under a sharp line—might require additional informational inputs. Consider the residency requirement a child must meet to qualify a taxpayer for the Earned Income Tax Credit (EITC). Under the current sharp line rule, the child must reside with the taxpayer for 183 days or more during the year. Under a sliding scale version of this rule, a taxpayer might qualify for a credit

66 IRC § 1361.
67 It would also generate many other complexities, such as how to treat entities where the number of shareholders changes during the tax year (perhaps the maximum, or a weighted average, could be used), or challenges related to consistency in treatment across years.
68 IRC § 152(c)(1)(B).
amount that varies in proportion to the number of days the child spends with the taxpayer. Under the sharp line, a taxpayer who lived with the child for fewer than 183 days would not be able to claim the child for the credit, and hence would not have to provide the other information required to determine the allowable EITC (at least assuming the taxpayer doesn’t qualify on some other grounds). In contrast, under the sliding scale, the taxpayer would have to provide other potentially relevant information for determining EITC, such as the breakdown between earned and unearned income.69

Finally, the above discussion indicates that the informational complexity of a sliding scale depends in part on the granularity of the classification variable. When the classification variable is coarser than a pure sliding scale (but still less coarse than a sharp line), the informational complexity can be alleviated. Considering the child residency example described above, a version of a sliding scale that would be less informationally complex would be to apportion the EITC based on the number of months during the year in which the child primarily lived with the taxpayer. This could reduce the amount of information a taxpayer would have to keep track of: If a child spent most of a month with the taxpayer, the taxpayer would not have to keep track of the precise number of days of shared residence.70

C. Implications for the Complexity of Tax Filing

Although sliding scales tend to be more complex than sharp lines (both computationally and informationally), the overall effect on the complexity of tax filing varies dramatically by context. Given that 95% of taxpayers rely on assisted preparation, computational complexity tends not to be a substantial drawback to using sliding scales today. Even if the tax law imposed a large number of sliding scales, the required calculations are trivial given the computing power of modern computers.

In contrast, even with assisted preparation methods, the role of informational complexity remains important.71 Recall that sliding scales are more informationally complex because they require more

69 See Goldin, note 64.
70 An example of a hybrid sharp line/sliding scale currently in the tax code is the formula for the dividends received deduction, which allows corporations a deduction for dividends they receive based on the degree to which they control the issuing corporation. That sliding scale is quite coarse because there are only three categories for the degree of control. See IRC § 243. Note that this provision resembles a sliding scale because the amount of deduction varies according to a nonbinary classification; it resembles a sharp line because the amount of deduction is fixed within a category.
Granular information on $x$ and because the additional tax regimes a taxpayer must apply might require additional information. The importance of both of these factors for tax filing varies significantly by setting. For most sharp lines that depend on the date of an event—such as childbirth, marriage, or the capital gains holding period—the extra informational complexity from converting a sharp line to a sliding scale is trivial or nonexistent, since the single date already contains all of the information required to calculate $x$. In other settings, where determining a taxpayer’s value of $x$ requires keeping track of many events—for example, days spent in a state during a year, or days a child resides with a taxpayer—the additional complexity of a sliding scale relative to a sharp line can be substantial. Cutting against this, however, is that taxpayers who are close to the threshold under a sharp line would already need to keep track of the classification variable with a high degree of precision.

Finally, in cases when a sliding scale does increase the complexity of the filing process relative to a sharp line, the overall effect on social welfare depends in part on whether the additional complexity deters taxpayers from filing a return in the first place. For most taxpayers with a filing requirement, we suspect this possibility is remote, since failing to file is often detectable by the IRS (at least for taxpayers with income reported on information returns), and the penalties for failing to file are nontrivial. However, for taxpayers without a filing requirement and who are owed a refund, it is possible that the additional informational complexity of a sliding scale will deter some taxpayers from filing, making them worse off by causing them to miss the refund they would otherwise receive.

D. Implications for the Complexity of Tax Planning

In addition to affecting the complexity of preparing one’s taxes, the policy choice between a sharp line and a sliding scale also affects the complexity of tax planning—that is, the process by which individuals

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72 Technological improvements might lessen these record-keeping costs in the future—for example, if more taxpayers use smartphones that can automatically record the taxpayer’s time spent in various states over the course of the year. See generally, Kathleen DeLaney Thomas, User-Friendly Taxpaying, 92 Ind. L. J. 1509 (2017) (discussing how the complexity of tax laws may encourage taxpayer dishonesty, as well as calling for the utilization of technology to simplify specific aspects of the taxpaying process).

73 See Stewart, note 39, for an extreme example of the record-keeping requirements a sharp line rule can impose on taxpayers.

74 One way to avoid this outcome could be to offer taxpayers an election between selecting the sliding scale and being taxed under the less advantageous regime. This would alleviate some of the welfare costs for taxpayers who would suffer most from the complexity of a sliding scale.
take taxes into account when determining their behavior. In general, tax planning grows more complex as it becomes more difficult for taxpayers to account for and predict how potential decisions would affect their tax liability. As with tax filing, the difficulty of making these determinations under a sharp line or a sliding scale depends on both computational and informational complexity.

First, to the extent a sliding scale is more informationally complex than a sharp line, tax planning under the former may be more difficult than under the latter. By increasing the information on which tax liability depends, switching from a sharp line to a sliding scale would make it harder for taxpayers to predict their exact tax liability under various choices of \( x \), even if they are using tax software for assistance. For example, consider the problem faced by taxpayers attempting to estimate their marginal tax rate when the state residency rule is a sliding scale, and when taxpayers split the year between states with different tax rates. Taxpayers might expect with high confidence to spend less than half of the tax year in a particular state (simplifying the task of predicting their marginal tax rate under the sharp line), but not be able to predict with confidence exactly how many days they will spend in each state, and hence, the degree to which each state’s tax rate will apply. In other cases, such as when selecting a date to get married or when to sell a stock, the additional informational complexity of a sliding scale is unlikely to make tax planning more difficult.

Turning to computational complexity, the question of whether a sharp line or a sliding scale better facilitates tax planning is ambiguous. On the one hand, the higher computational complexity of a sliding scale makes it more difficult for taxpayers to calculate their precise tax liability under any potential choice of \( x \). However, the practical importance of this concern is mitigated by the fact that estimating one’s tax liability ahead of time with any degree of precision is already quite difficult, even when only sharp lines are involved. Taxpayers who care about making an accurate prediction would probably rely on software, even absent a sliding scale. And as discussed above, sliding scales tend not to meaningfully increase the computational complexity experienced by taxpayers who use software. Hence, smoothing a sharp line into a sliding scale would not usually make tax planning much more complex.

In addition, a sliding scale could actually simplify tax planning in some cases, by making it easier for taxpayers to understand the tax implications of the choices they make. In particular, it is often easier to understand how changes in \( x \) affect tax liability under a sliding scale, because understanding the relationship between tax liability and \( x \) does not require predicting whether \( x \) will cross a particular threshold.
For example, under the sliding scale approach to the EITC child residency requirement, it is easy for taxpayers to understand—in general terms—that the more days the child lives with the taxpayer, the larger the taxpayer's EITC. In contrast, understanding the tax implications of the taxpayer spending one more or one less day with a child under the sharp line version of the rule requires that the taxpayer learn both where the line is (i.e., at 183 days), as well as predict the number of other days during the year the child will spend with the taxpayer.75

Apart from differences in the cost of tax planning associated with a sharp line and a sliding scale, the two instruments also differ in the incentives they generate for taxpayers to undertake tax planning in the first place. We return to these considerations in Part VI below.

IV. FAIRNESS AND CLASSIFICATION CONSIDERATIONS

In this Part we focus on considerations related to horizontal equity, and more generally, the rationale for imposing different taxes on individuals with different values of the classification variable. Because the rationales for differential taxation can vary dramatically across contexts, our discussion in this Part is necessarily general.

Our main claim is that when there is a reason to differently tax individuals with \( x = 1 \) versus \( x = 0 \), it will usually be the case that the same reason implies that taxpayers with intermediate values of \( x \) (between 0 and 1) should be taxed somewhere between those with \( x = 0 \) and those with \( x = 1 \). This issue has been discussed outside of the tax context,76 and its application to tax is straightforward. As one influential treatment of the subject has put it: “Smooth relationships often do a better job of preserving morally relevant information than do bumpy relationships.”77

There are two steps to our argument. First, note that most reasons for differential taxation across taxpayers have to do with considerations that vary in degree, rather than in kind. For example, taxpayers might be treated differently by the tax law because they differ in their ability to pay. Ability to pay exists upon a continuum; someone who earns $10,000 a year has less ability to pay a $1000 tax than someone who is otherwise identical but earns $20,000 a year, and the latter has less ability to pay the tax than someone who earns $30,000 a year. Similarly, under a Mirleesian optimal tax, otherwise identical

75 Put differently, to the extent taxpayers make tax planning decisions on the margin, a sliding scale will likely be easier for taxpayers to understand than a sharp line because the marginal effect of a change in \( x \) does not depend on the level of \( x \) relative to \( x \).
76 See, e.g., Alexander, note 7 (discussing this issue in the philosophical context).
77 Kolber, note 7, at 657.
individuals may be taxed at different rates based on the elasticity of their labor supply, and the elasticity of labor supply falls along a spectrum.

The second step of the argument is to note that when taxpayer characteristics can be ordered along a spectrum (i.e., when they can be represented by a variable like \( x \) in our model), the relationship between the characteristic and the underlying objective is typically continuous. That is, small changes in the characteristic translate into small changes in the degree to which the underlying objective is satisfied. For taxes that vary based on ability to pay, small changes in income would typically translate into small differences in ability to pay. For taxes that vary based on the elasticity of labor supply, small differences in elasticities would typically translate into small differences in tax rates.

These claims suggest that if tax liability is to depend on a classification variable, the relationship should be continuous—that is, small changes in the classification variable should translate into small changes in tax liability. A sharp line fails this requirement: Near the line, small changes in the classification variable imply large changes in tax liability. Away from the line, small changes in the classification variable do not result in any change in tax liability at all. In contrast, the sliding scale maps small changes in the classification variable to small changes in tax liability.

The arguments in this Part imply that tax liability should depend continuously on the classification variable, but do not necessarily imply that the relationship should be linear (as under a sliding scale). In settings where the relationship between the underlying classification rationale and the classification variable is discontinuous or highly nonlinear, a sharp line might do a better job approximating this relationship than a sliding scale.

To illustrate, consider a transaction in which a corporation redeems its stock from a shareholder. Depending on the circumstances, the transaction may either be taxed as if the shareholder exchanged its stock with a third party or as if it received a corporate distribution. Section 302(b)(2) specifies that the transaction will be taxed as an

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78 To take a different example, child tax benefits are often justified based on differences in ability to pay between taxpayers with and without children. However, the head of household filing status contains a sharp line based on the number of children a taxpayer claims; a taxpayer must claim at least one child to qualify for the benefit, but the benefit amount does not increase in the number of children claimed. Because supporting additional children tends to reduce a taxpayer’s ability to pay, smoothing out the benefit into a sliding scale could better further the goals of the provision. See Jacob Goldin & Zachary Liscow, Beyond Head of Household: Rethinking the Taxation of Single Parents, 71 Tax L. Rev. 367 (2018).
exchange if the distribution is "substantially disproportionate" with respect to the shareholder, which requires, among other things, that the shareholder ends up owning less than 50% of the corporation's total voting power following the transaction's completion. If one of the goals of the provision is to distinguish between transactions that affect shareholders' control of the corporation, employing a sharp line at 50% makes sense, since the relationship between voting power and control is itself discontinuous at 50%. Although it is possible to imagine other examples where the underlying objective of a provision is discontinuous, it is difficult to think of many practical applications in which this is likely to be the case.

A related consideration is that sharp lines tend to violate basic notions of fairness and horizontal equity by treating differently taxpayers who happen to fall on opposite sides of the line, but who are otherwise quite similar. For example, under the sharp line in current law, a child born at 11:59 p.m. on December 31 can qualify the parent for the full year's Earned Income Tax Credit (up to $3400 in 2017 for a parent with one child); but if the child is born just a few minutes later, the parent would, at most, qualify for the childless EITC, with a maximum credit of only $510. Because two otherwise identical parents, with children born only minutes apart, appear alike in all normatively relevant dimensions (such as ability to pay), it seems arbitrary and unfair for their tax liabilities to differ by thousands of dollars. In contrast, under a sliding scale, in which each parent could claim the credit in proportion to the fraction of the tax year that elapses after the child is born, each parent could claim a similar amount of EITC for the year. Finally, note that the unfairness of a sharp line can be exacerbated in its effect on welfare through individuals' psychological aversion to "near misses," which sliding scales tend to avoid.

V. Tax Administration Considerations

This Part considers the relative ease of administering a sharp line versus a sliding scale. By administrative costs, we mean costs incurred

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79 IRC § 302(b)(2)(B).
80 IRC § 32(b); Rev. Proc. 2016-55.
81 Of course, the parent of the child born in the new year might qualify for an extra year of EITC nineteen to twenty-four years later, when the older child has aged out. IRC § 152(c)(3). This later benefit is reduced by the time value of money and the possibility that the parent's income will have changed by the later date so that he or she no longer qualifies for the credit.
by the taxing authority—unlike complexity costs, which are borne by individual taxpayers.

One type of administrative cost associated with sliding scales is their enhanced informational requirements relative to sharp lines. In this respect, the considerations parallel the difference in informational complexity between sharp lines and sliding scales; the latter require the taxing authority to collect more granular information about the classification variable, and may also require additional information to be collected for purposes of calculating a taxpayer’s liability under a second tax regime.

How do these additional informational requirements affect administrability? As with informational complexity for the taxpayer, the practical importance of these considerations varies greatly by setting. If an information reporting regime is already in place, there may be little incremental cost to expanding the system to cover additional taxpayers (although there may be costs on other parties, like employers, from expanding such a system). Similarly, the finer granularity of the information required under a sliding scale will not much increase administrative costs when the information can be derived from other information that is easily available. For example, Form 1040 already asks for the birthdays of children claimed on the tax return, even though all that is technically required by any provision is whether the child’s age is above or below some threshold.

A second consideration for assessing administrative costs under the two types of instruments is that sliding scales can reduce the stakes of many enforcement determinations, and correspondingly, the amount of taxpayer pushback such determinations provoke. To illustrate, consider an analogy from a very different context. In soccer, a foul committed just outside the penalty box has a much lower likelihood of leading to a goal than a foul committed just inside the penalty box.83

Because of this large discontinuity in the location of where the foul occurred, soccer referees have turned to video assisted review, which has been criticized for slowing the pace of the game. If the rule for assigning penalty kicks were replaced by a sliding scale, there would be less pressure on referees to determine exactly where the foul occurred

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83 A foul committed outside the penalty box will result in a free kick at the spot of the foul, at least eighteen yards from the (center of the) goal and will sometimes present a poor shooting angle. By contrast, a foul committed just inside the box will always result in a penalty kick from twelve yards away from the goal, dead center. As a result, a foul just inside the box leads to a goal about 80% of the time in World Cup games, compared to a foul just on the other side of the line, where even from the most favorable positions, one study found that less than 25% of free-kicks turn into goals. See Stan Ivanov, Penalty Kick Statistics at FIFA World Cups—Expected Goals, http://intelligentfc.com/penalty-kicks-expected-goals-at-fifa-world-cup/; Daniel Link et. al, A Topography of Free Kicks in Soccer, 34 J. Sports Sciences 2312, 2316 (2016).
and video replay as to the exact location of the foul would not be necessary.

Finally, a sliding scale might make it harder for the taxing authority to target enforcement efforts where they are most likely to be effective. With a sharp line, there is typically little revenue to gain by auditing taxpayers' reported values of $x$ for taxpayers who are far from the line (in either direction), at least in cases where it is more difficult to cheat by a lot than it is to cheat by a little. In contrast, with a sliding scale, there will generally be some incentive for all taxpayers to cheat, making it harder for the tax authority to identify high potential returns to audit.

VI. TAX PLANNING CONSIDERATIONS

This Part considers how the choice between sliding scales and sharp lines affects taxpayers' propensity to engage in tax planning. By tax planning, we mean the process by which taxpayers account for taxes in deciding on their behavior. From a societal perspective, tax planning is generally undesirable for two reasons. First, to the extent it is costly for taxpayers to engage in, it diverts resources that could have gone to other productive uses. Second, when taxpayers adjust their behavior to reduce their tax liability, they generate deadweight loss, which reduces the efficiency of the tax. Against these factors, failing to plan causes taxpayers to make privately suboptimal decisions, which also factors into social welfare.

To understand how sharp lines and sliding scales might affect tax planning, we consider a "bounded rationality" model of decision-making. Specifically, we assume that it is costly for taxpayers to account for the tax when making their decision about $x$. These costs may be financial—such as the opportunity cost of time spent thinking about tax considerations or the fees charged by tax lawyers or accountants—or nonfinancial, such as the unpleasantness of engaging in difficult calculations or thinking about taxes in the first place. We assume taxpayers decide whether to engage in tax planning by comparing the expected benefits of accounting for the tax in their decision-making to the costs of doing so. As we describe below, the choice between sliding scales and sharp lines can potentially affect both the benefits and costs sides of this trade-off.

84 Or, similarly, taxpayers may have a disproportionately strong moral aversion to understating their tax liability by a lot as opposed by a little.
With respect to the costs of tax planning, our discussion in Part III indicates that either a sharp line or a sliding scale can make tax planning costlier. Sharp lines are less computationally complex—making it easier for taxpayers to estimate their precise tax liability—but the question of how changes in one's behavior affect tax liability may be easier to answer under a sliding scale. Hence, depending on the situation, smoothing a sharp line into a sliding scale may either increase or decrease the costs of tax planning.

In addition, the choice between sharp lines and sliding scales also affects the benefits of engaging in tax planning. Again, however, which approach produces a larger benefit depends on the setting. Consider, for example, an individual whose ideal value of x is just to the right of the sharp line cutoff $\bar{x}$. Under the sharp line, such an individual faces a large incentive to take taxes into account when choosing a value of x. For this taxpayer, the sharp line creates a large benefit to tax planning. In contrast, taxpayers who are confident that their value of x is far from the cutoff will have little reason to account for the tax under a sharp line, because considering taxes is unlikely to cause them to change their behavior.

Under the sliding scale, the incentive to engage in tax planning is evened out among taxpayers. For taxpayers who were confident they were far from the cutoff under a sharp line, the incentive to account for the tax is increased under a sliding scale. In contrast, taxpayers who were near the line under a sharp line face a much smaller incentive to account for the tax under the sliding scale because there is less scope for them to reduce their tax liability by adjusting their behavior. As with efficiency, therefore, the question of which type of instrument results in a net increase in the benefits of tax planning depends in part on the distribution of taxpayers' ideal points.

Overall, it is difficult to draw general conclusions about whether the sharp line or sliding scale results in more tax planning. Depending on the situation, either instrument may increase or decrease the costs of tax planning, and similarly, either instrument may increase or decrease the benefits. The net effect of these considerations is therefore likely to vary based on the setting.

**VII. ADDITIONAL CONSIDERATIONS**

This Part considers a range of additional considerations that affect the policy choice between sharp lines and sliding scales.
A. Standards versus Rules

Up to this point, we have focused on the choice between sharp lines and sliding scales in the case of rules, where the legal outcome is a known determinative function of a set of specified conditions. In this Section, we extend our analysis to the case of standards, where the legal outcome follows from applying a general principle. To preview our results, we argue that the case for sharp lines is stronger when the test under consideration is a standard. In such cases, the efficiency differences between sharp lines and sliding scales tend to be less pronounced, and the administrative and complexity advantages of sharp lines tend to be more extreme.

Like rules, standards can be designed either as sharp lines or as sliding scales. The reasonable person concept underlying negligence torts is a classic example of a sharp line standard. Consider defendants whose actions cause damage to plaintiffs. If defendants’ level of care even slightly exceeds that of reasonable people, they are not negligent, and therefore have no tort liability. If defendants had been slightly less careful, however, they could face liability for the full amount of the plaintiff’s damages. In contrast, a sliding scale version of the negligence standard might assign liability to the defendant for the plaintiff’s damages based on the degree to which the defendant took care.

Turning to tax law, it is easy to imagine possibilities for smoothing out some of the sharp line standards in place today. For example, to determine whether a transfer between taxpayers constitutes a gift, the key question is whether the transferor’s motivation was sufficiently “detached and disinterested.” If so, the recipient of the transfer can exclude it from income; if not, the recipient must include it in income. In theory, the current sharp line standard could be replaced by a sliding scale standard, so that if the transferor’s motivation was, say, 70% detached and disinterested, the recipient would include 30% of the transferred amount in income. Similarly, the sharp line test that currently divides employees from independent contractors could be replaced by a sliding scale, in which treatment would depend on the degree to which a work relationship satisfies the various factors that form the current test.

In considering the choice between sharp line standards and sliding scale standards, several aspects of the analysis are worth keeping in mind. First, as mentioned above, standards often generate ex ante

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86 Seemingly against this claim, David Weisbach argues that rules create discontinuities in the law whereas standards generate continuous legal treatment. See Weisbach, note 60. From Weisbach’s discussion, however, it is clear that the only form of rule that he is considering is a sharp line. Our analysis builds on his by allowing for classification tests that provide certainty to taxpayers while also being continuous in the incentives they generate.
uncertainty for taxpayers, in the sense that taxpayers cannot predict with complete accuracy how their choices will shape their tax liability. As discussed in Part II.F, in settings with uncertainty, the behavioral incentives generated by sharp lines more closely resemble those generated by sliding scales.\(^8\) Hence, efficiency considerations tend to be less important when choosing between sharp lines and sliding scales in the case of a standard relative to a rule.

Another relevant consideration is that the factors that determine how taxpayers are classified under a standard can be difficult to quantify.\(^8\) This matters because under a sliding scale, tax liability depends on the exact value of the classification variable, not simply on whether it exceeds a particular threshold. For example, in considering whether a transfer should be considered a "sham" for tax purposes, how would a taxpayer or the tax authority attempt to quantify the degree to which it was or was not a sham? Of course, this quantification problem also arises under a sharp line, since one must compare the classification variable to the relevant threshold, but the additional demands of the sliding scale to precisely quantify the classification variable for all taxpayers can exacerbate these difficulties.

Finally, a frequently cited benefit of standards over rules in the tax context is that they are less conducive to aggressive tax planning.\(^8\) Under a (sharp line) rule, the argument goes, taxpayers can walk right up to the line without crossing it to reduce their tax liability by the maximum amount allowable. In contrast, under a (sharp line) standard, the precise location of the line is unclear, which means that more aggressive transactions run a greater risk of receiving dis-favorable tax treatment. Because sliding scale rules also smooth the tax liability associated with taking aggressive positions, they also reduce the incentives to engage in aggressive tax planning. Unlike sharp line

\(^8\) Similarly, this perspective suggests modeling settings characterized by both a standard and a safe harbor and/or sure shipwreck as a Z-shaped expected liability function, where expected liability rises smoothly over intermediate values of \(x\), but jumps discretely at the level of the classification variable that the safe harbor or sure shipwreck kicks in. For an analysis of the incentives faced by taxpayers in such settings, see Morse, note 41, at 1399-1402 & fig. 1; see also Raskolnikov, note 60, at 509-13 (discussing the behavioral effects of settings characterized by changes in the slope of the relationship between taxpayer behavior and expected tax liability).

88 Indeed, part of what makes a standard a standard is the law's unwillingness to specify ex ante what the dispositive factors will be. Examples of this are common in tax. For example, the Ninth Circuit applies a nonexclusive, unweighted, eleven-factor test for distinguishing debt and equity for tax purposes. The Ninth Circuit recently observed that "while such a free-floating inquiry is hardly a paragon of judicial predictability, it's the necessary evil of a tax code that mistakes a messy spectrum for a simple binary, and has repeatedly failed to offer the courts statutory or regulatory guidance." Hewlett-Packard Co. v. Commissioner, 875 F.3d 494, 498 (9th Cir. 2017).

89 Weisbach, note 60.
standards, however, they do so without creating uncertainty for taxpayers. Hence, sliding scale rules may be particularly desirable in settings in which taxpayers are risk averse and in which the government is concerned with preventing aggressive tax planning.

B. Intent-Based Classifications

One special category of rules and standards that are prevalent throughout the tax code are those that turn on an individual’s intent or purpose in undertaking some activity. Some prominent examples include the “primary business purpose” test for certain allowable business expense deductions, the “detached and disinterested generosity” motivation for classifying a transfer as a gift, and the subjective intent prong of the “economic purpose” test. Each of these classifications is currently structured as a sharp line; treatment is all-or-nothing depending on whether the requisite test is satisfied.

As with other classifications, one could imagine smoothing out the sharp lines governing intent-based classifications by replacing them with sliding scales. For example, an otherwise-deductible business expense that was 49% motivated by business considerations cannot be deducted at all under current law, since the business motivation was not the taxpayer’s primary purpose, but 100% of the expense would be deductible if instead 51% of its purpose was business-related. A sliding scale version of this rule would treat these cases similarly, with 49% of the expense being deductible in the first instance and 51% in the second.

In thinking about the choice between sharp lines and sliding scales in the context of intent-based classifications, some but not all of the previously discussed considerations apply. On the one hand, the rationale underlying the classification, as well as fairness considerations, will typically support the sliding scale, as they do in other areas of the law. Someone who undertakes an expense that is 51% business-motivated is similarly situated (with respect to the objectives of the statute) to someone who undertakes the same expense, but who is slightly more motivated by nonbusiness considerations. The sliding scale respects this similarity, whereas the sharp line does not.

On the other hand, the efficiency difference between sliding scales and sharp lines is muted in the case of intent-based classifications. This is because the variable underlying intent-based classifications is not about the taxpayer’s behavior—at least in most cases, one cannot

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simply choose one's motivation behind an action. That is, consider taxpayers who undertake an expense for 51% personal reasons. To reduce their tax liability, they would prefer to reduce the personal motivation to only 49%, but unlike the other settings we have considered, they do not have the ability to do so. Put differently, taxpayers cannot adjust the motivation with which they do something to recognize a tax benefit; otherwise, every taxpayer would simply choose the tax-preferred motivation! Because taxpayers do not choose their motivation, intent-based classifications do not distort behavior in this respect. On the other hand, because the taxing authorities and courts must infer motive primarily from objective indicia like how the taxpayers spent their time on a trip that had business and personal aspects, the efficiency analysis sometimes applies well even to intent-based classifications like primary business purpose.

Finally, as with standards, intent-based classifications are challenging to implement as sliding scales because the classification variable (i.e., the taxpayer's purpose) is difficult to quantify. What does it mean for a taxpayer's purpose in taking some action to be 49% personal versus 51% personal? Although it might be possible to develop an approach for quantifying motivations in such a way that would allow a sliding scale, doing so would likely be costly for taxpayers and the tax authority to implement. A more practical alternative to a sharp line in this setting might therefore be a granular sliding scale, with multiple discrete categories (e.g., one's purpose might be mostly business, mostly personal, or both business and personal).91

C. Democratic Legitimacy and Privacy

A potential objection to sliding scales is that, by increasing the complexity of the tax system, they make it less transparent, and a transparent tax system is important for democratic legitimacy. We think this concern is misplaced. Practically speaking, the relationship between taxpayers' decisions and their tax liability is already quite obscure, even under the current sharp-line-dominated system. It seems unlikely that switching to a sliding scale would further reduce transparency in any meaningful sense. If anything, a sliding scale might actually be more transparent because it makes the relationship between taxpayers' decisions and their liability more intuitive. Even if

91 One way to think about the problem here is that sometimes it is not actually sensible to treat the underlying classification variable as continuous between 0 and 1. That is, it might be possible to determine that some activities have more business-purpose motivation than others, but still be unable to assign each activity a precise fraction for how much business purpose it had, at least in any meaningful way.
taxpayers do not know the precise amount by which slightly increasing \( x \) will affect their tax liability, they can understand the qualitative relationship under a sliding scale, and at least the direction of the effect. In contrast, under a sharp line, taxpayers must understand where they fall relative to the line to understand how their choices will affect their liability.

By requiring greater granularity in information, sliding scales may sometimes raise privacy concerns not present with a sharp line, particularly with respect to where taxpayers (or their dependents) spend their time. We think that—at least for nearly all practical sliding scale proposals—this potential issue is fairly limited. Taxpayers who split their time between states or countries already often must report how many days they spent in each place.\(^92\) Adopting sliding scales of the kind we examine below would expand this requirement to a greater number of individuals, but we think it is still fairly coarse information compared to, for example, what is routinely given over to a taxpayers’ cell phone company.

\[ \text{D. Classifications Based on Income} \]

This Section briefly considers special issues that arise with respect to classifications based on income. In most areas of tax law where liability varies by income, the conventional wisdom is that gradual phase-ins and phase-outs are preferable to sharp lines (or cliffs, as they’re often referred to in this setting) because of the high marginal tax rates generated by the latter. The tax code mostly reflects this conventional wisdom—provisions that depend on income typically do so in a smooth manner—but there are exceptions.\(^93\)

For the most part, our analysis supports the conventional wisdom that sharp lines in income are undesirable. From an efficiency standpoint, our results imply that a sharp line is less distortive than a sliding scale only when there is a dip in the distribution of taxpayers’ ideal, pretax incomes near the point that the sharp line is drawn.\(^94\) Although it is possible that social or regulatory factors might produce

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\(^92\) See, e.g., NY State Tax Form IT-203-B.

\(^93\) Most importantly, the tax rates imposed by § 1 impose tax liability as a continuous function of taxable income, although frequent confusion by elected officials and the media about the difference between marginal and average tax rates suggests this point is not always appreciated. See IRC § 1. In a different context, the industry and wage limitations for the new § 199A deduction phase in gradually by income. See IRC § 199A. In contrast, the unearned income limitation on EITC recipients operates as a sharp line. For many more examples, see generally Viswanathan, note 15.

\(^94\) By “ideal,” recall from Part II that we are referring to the amount of income the taxpayer would choose to earn absent tax considerations.
such a dip in the distribution (because of a minimum wage law, for example), this seems unlikely in most cases.

Along the same lines, the factors that sometimes make a sliding scale more informationally complex than a sharp line do not apply when the classification variable is income. Even without the sliding scale, taxpayers must already keep track of and report their precise income (down to the nearest dollar) to the tax authority for purposes of computing their tax liability. Hence, converting an income-based sharp line into a sliding scale would not require any new information.

Finally, because the role of income-based requirements in tax law is often to track ability to pay, and because ability to pay usually varies smoothly in income, the underlying goals of the provision would also typically support sliding scales when it comes to income-based classifications.

E. Taxes Designed to Shape Behavior

A variety of real-world tax provisions are designed to deter behavior that has negative externalities, and similarly various credits and deductions are intended to promote behavior with positive externalities. For example, § 162 denies deductions for the cost of lobbying to help prevent the "spread [of] insidious influences through legislative halls." The ban on such deductions applies even when the lobbying expenses are solely for the production of income and can be thought of as equivalent to a special tax on lobbying expenses levied at the taxpayer's marginal tax rate.

If the goal of a tax was solely to encourage or discourage some behavior by as much as possible (at a fixed level of revenue), the optimal policy comparison between sliding scales and sharp lines would proceed as in Part II. That is, whether sharp lines or sliding scales induce a larger change in taxpayer behavior depends on the distribution of taxpayers' ideal points—how they would choose to behave if the tax did not exist. For example, when taxpayers' ideal points are evenly distributed, the sliding scale induces a smaller response from a larger share of taxpayers, whereas the sharp line induces a large response from taxpayers close to the line. Thus, when the goal of a provision is simply to affect behavior by as much as possible, our prior analysis highlights how policymakers should proceed.

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96 When the outcome associated with crossing the sharp line is to incur a penalty or reward, there might be an additional incentive effect from taxpayers desiring to be on the "good" side of the line. A sliding scale would lack this additional motivational force.
However, the efficiency analysis differs when the goal of a tax is not simply to affect behavior by as much as possible, but rather to account for some positive or negative externality. If every dollar spent on lobbying creates the same harm—say $H$—to the political process, for example, then a sliding scale tax on lobbying expenses equal to $H$ per dollar of lobbying will not create any deadweight loss. Instead, while firms and individuals facing this tax will cut down on the amount of lobbying they do relative to their private ideal points, the tax will make them account for the marginal social harm of such lobbying and will lead to the most efficient outcome.

In contrast, a lobbying tax that takes the form of a sharp line cannot induce the efficient outcome, at least when the location of the line does not vary by taxpayer. Intuitively, efficiency requires that each taxpayer account for the social costs and benefits of their behavior when choosing $x$, but only some taxpayers are induced to do so under the sharp line. On the other hand, when a line’s location can vary by taxpayer, sharp lines can be set to induce each taxpayer to select the socially optimal value of $x$.

F. Multifactor Tests

In some cases a tax classification depends on multiple factors. The form of a multifactor test affects the range of possibilities for sharp lines versus sliding scales.

First, consider a multifactor test where some or all of the individual factors are themselves binary. In such cases, it might be possible to smooth out the underlying factors themselves. For example, one of the factors in the test for characterizing individuals as employees versus independent contractors is the amount of control individuals have over

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97 This is the classic "Pigouvian" tax. See, e.g., James M. Sallee & Joel Slemrod, Car Notches: Strategic Automaker Responses to Fuel Economy Policy, 96 J. Pub. Econ. 981, 991 (2012) (observing that a Pigouvian tax equal to the marginal harm will be fully efficient socially)

98 The social harm may differ, though, if it is based on other taxpayer attributes in addition to the amount of lobbying the taxpayer does. In that case, the analysis is more complicated. Accordingly, a sharp line may be more efficient than a linear sliding scale if the former more closely resembles the optimal nonlinear Pigouvian tax schedule.

99 Those taxpayers who jump to $\bar{x}$ are in a sense overcompensating for the harm of lobbying and the remainder who do not move in response to the sharp line do not account enough for the harm they are causing. Formally there is deadweight loss because taxpayers who jump to $\bar{x}$ could engage in a Pareto improving trade: Those at $\bar{x}$ would pay the non-movers to slightly decrease their lobbying and allow those at $\bar{x}$ to lobby a bit more. The total amount of lobbying would be unchanged, but the additional amount of lobbying is more valuable to those at $\bar{x}$ because they are far from their private ideal point. Such a Pareto improving trade cannot be made under the sliding scale. See Sallee & Slemrod, note 97, at 991-94 (providing formal demonstration of the welfare costs of these uneven incentives).

100 Martin L. Weitzman, Prices vs. Quantities, 41 Rev. of Econ. Stud. 477 (1974).
how they perform their work. Suppose an individual exercises just enough control that the factor comes out in favor of independent contractor status. Alternatively, suppose individuals exercise near-total control over how they perform their work. Under a sharp line, these two cases would equally support the conclusion that the individual should be classified as an independent contractor rather than an employee. In contrast, a sliding scale would treat this factor as more strongly supporting independent contractor status in the second case than in the first.

A second possibility for smoothing out a multifactor test is to smooth out the relationship between the individual factors and the legal result that they determine. Suppose classification depends on two factors, $y$ and $z$. Let $u$ represent an aggregation of $y$ and $z$ with weights corresponding to the multifactor test, such that $u$ is increasing in $y$ and $z$ and $u > \bar{u}$ if and only if the high tax regime applies. Note that as described, this multifactor test implements a sharp line based on $u$, and therefore based on $y$ and $z$ as well, where the location of the line for one factor depends on the value of the other factors as well. As with single-factor tests, this multifactor test can be smoothed out by assigning tax liability in proportion to $u$, rather than based on whether or not $u$ crosses the stated threshold. One simple version of this, when the factors that enter into the test are binary, is to assign tax liability in proportion to the number of individual factors that are satisfied.

How do the efficiency properties of sharp lines and sliding scales differ in the context of multifactor tests? First consider the question of whether to smooth out an individual factor that enters into a larger test. The behavioral consequences of a sharp line here are similar to those in the single-factor test setting; the main difference is that the tax implications of crossing the line for one factor hinge on the resolution of the other factors. When the factor in question is determinative, the tax implications of crossing the line—and hence the deadweight loss generated—will be particularly large. When the factor in question is unlikely to be determinative, the incentives to account for the line will be muted. Hence, the efficiency gains or losses from smoothing out the sharp line are the same as in the single-factor case, but the stakes vary depending on how likely it is that the factor in question will be determinative.

Turning to the question of whether to smooth out the relationship between the factors and tax liability, consider how the smoothness of this relationship affects taxpayer decisions that relate to the factors. Under a sharp line, the incentives to adjust one factor depend on the resolution of the other factors as well. For example, when a taxpayer's ideal values of $y$ and $z$ imply a value of $u$ that is just above $\bar{u}$, the
taxpayer would face strong incentives to adjust \( y \) and \( z \) to lower \( u \). The deadweight loss that results would depend on which factor the taxpayer adjusts, and the costs of making that adjustment.\(^{101}\) Under the sliding scale, in contrast, all taxpayers face an incentive to reduce \( u \) by adjusting \( y \) and \( z \), but these incentives are smaller than those faced by taxpayers near the cutoff under the sharp line. Hence, determining whether the sliding scale or the sharp line is more efficient for the multifactor test is similar to the analysis for a single-factor test, but the key question concerns the distribution of \( u^* \), which in turn is based on the distribution of taxpayers' ideal points for the underlying factors.

The analysis thus far has assumed that taxpayers and the tax authority know the aggregation of individual factors demanded by the multifactor test, so that both sides can calculate \( u \). In practice, multifactor tests are usually quite vague about how to aggregate individual factors into an overall legal determination. Thus, in the presence of such tests, the relationship between taxpayer behavior and tax liability can be characterized by substantial uncertainty. As with single-factor tests in the presence of uncertainty, the efficiency consequences of the choice between sliding scales and sharp lines for multifactor tests are therefore somewhat muted.

From a complexity and administration perspective, one potential upside to a sharp line approach for multifactor tests is that it sometimes allows taxpayers and the tax authority to ignore those factors that are not determinative. For example, if under a sharp line multifactor test, the value of one factor is so extreme that the overall test is likely to come out in a particular direction, determining tax liability does not require resolving the remaining factors. In contrast, under a sliding scale, calculating tax liability requires resolving each of the potentially relevant factors. Similarly, determining liability under a sliding scale can require a more precise specification of how different factors are aggregated to reach a particular result than under a sharp line, which can be costly for the tax authority to articulate and complicated for taxpayers to learn about and apply.

VIII. EXAMPLES

This Part applies our framework to a number of examples from different areas of tax law.

\(^{101}\) Note that this analysis holds when the factors are binary, in addition to when they are continuous; when they are binary, adjusting behavior so that \( u \) falls below the line may be costlier because the change is more significant.
A. Child Tax Benefits

Our first example relates to tax benefits for taxpayers with children, such as the Child Tax Credit, the EITC, and the head of household filing status. For each of these provisions, the basic definition of a qualifying child comes from § 152(c) of the Internal Revenue Code, which has been interpreted to encompass children born at any point during the tax year. This rule embodies a sharp line: A child born on December 31 can qualify the taxpayer for the full year’s child tax benefits, but a child born the next day, on January 1, cannot. The stakes are significant; for low-income taxpayers, it could mean a difference of $5000 or more in their tax refund for the year.

Converting the child birth date sharp line into a sliding scale would be straightforward: Child tax benefits for newborns would be proportional to the fraction of the year elapsed after the child’s birth. For example, a child born on January 20 in a non-leap year would qualify for 345/365 of the benefit, since 345 days elapse during the year after the child’s birth. A child born on December 31 would qualify for 1/365 of the benefit. Note that the reform eliminates the year-end discontinuity; the taxpayer whose child is born on January 1 of the subsequent year would qualify for 0/365 of the benefit.

The considerations discussed earlier in the Article support converting the child’s birth requirement into a sliding scale. The efficiency benefits are likely to be positive but small. We have no reason to expect the sharp line at December 31 corresponds to a dip in the pretax distribution of ideal dates when parents would like to have their children. Moreover, there is evidence that a small number of parents react to the existing sharp line at December 31 by moving up planned C-sections or induced labor. But the deadweight loss from these changes is likely to be small assuming that most parents and medical professionals would be unwilling to speed up delivery dates by more than a week or two at most, regardless of the tax benefits.

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103 Presumably, the age limits would be reformed in parallel so that a child would qualify the taxpayer for a fraction of the otherwise-allowable benefit in the year the child aged out of eligibility.


105 It is also possible that some parents try to time conception so their child will be born in November or December rather than January or February. Here, however, the taxpayers’ substantial lack of control over the exact date of conception and birth will blunt the efficiency costs of using the sharp line.
true that having a sliding scale would create a general incentive for all children to be born slightly earlier in the year, but it would dramatically lessen the incentive under current law to hasten delivery for kids who would otherwise be born just after the start of the new year. Because the potential tax savings from shifting the delivery date by a couple of days or even weeks is relatively small under a sliding scale, we would not expect it to generate much in the way of deadweight loss.

Turning to classification considerations, here the case for a sliding scale is much stronger. The most frequently cited rationale for the child tax benefits is that having children reduces the taxpayer’s ability to pay. All else being equal, a taxpayer with a child born on December 31 has a very similar ability to pay as a taxpayer with a child born the next day, on January 1. Hence, smoothing out the tax benefit in proportion to the fraction of the year that the taxpayer supports a child will better align the tax benefit with its rationale.

Finally, although sliding scales can sometimes be more complex than sharp lines, this would not be the case for child tax benefits. In particular, the extra computational complexity here is unimportant, since it is easy for tax software to scale the size of the benefit based on the child’s birthday. Similarly, a sliding scale would not increase informational complexity because the extra information (the child’s birthday) is (hopefully) easy for parents to provide.

Overall, our arguments support smoothing the child tax benefits birth requirement from a sharp line into a sliding scale. There is little downside in terms of additional complexity, some modest but uncertain benefit in terms of efficiency, and a substantial upside in terms of furthering the ability to pay goals that underlie the provisions.

B. Long-Term Capital Gain Holding Period

Current law distinguishes between the sale of “long-term” and “short-term” capital assets, with gains from the former subject to a preferential tax rate.\(^\text{106}\) If the taxpayer holds the asset for at least one year, it is long term; otherwise it is short term. Today, the maximum tax rate on long-term gains is 20%, whereas the maximum tax rate on short-term gains is 37%, as with ordinary income.\(^\text{107}\) This classification test is a sharp line: Gains from the sale of a capital asset held for exactly one year are taxed at a much higher rate than gains from an asset held one day longer.\(^\text{108}\)

\(^{106}\) IRC §§ 1(h), 1222.
\(^{107}\) IRC §§ 1(h), 1222.
\(^{108}\) Under significantly older law, there was something closer to a sliding scale: “In 1934, for instance, Congress provided for a decreasing percentage of gain to be taxable the longer
How might the holding period line be converted into a sliding scale? Because assets can be held indefinitely, a sliding scale would operate over a subset of possible holding period lengths, such as a six-month transition period (say between nine months and fifteen months) or a one-year transition period (say between six months and eighteen months).

Beginning with efficiency, the concern with a sharp line is that it provides a large incentive for taxpayers to avoid selling marginal assets when they have held the asset for slightly less than a year. The simple model we proposed in Part I does not entirely fit this example, because taxpayers do not choose a holding period for their assets at a single point in time, but rather make sequential decisions about whether to hold or sell the asset on any given day. On any day, taxpayers will consider whether to hold or sell an asset based in part on their expectations about its risk and return. If taxpayers are considering whether to sell some asset after, say, an eleven-month holding period, under the sharp line they will consider not only the asset’s expected risk and return, but also the probability that if they do not sell the asset, they will continue to hold it until the one-year mark, at which point they secure the favorable tax treatment. The closer in time to the one-year mark, the greater the tax incentive to continue holding the asset. In contrast, farther from the one-year threshold—say after holding for only one month—the sharp line exerts less force because there’s a higher probability the taxpayer will decide to sell at some future date prior to the one-year threshold.

In terms of magnitude, we expect moderate deadweight loss from the sharp line. On the one hand, many capital assets are sold well before one year or well after the one-year mark, even with the current tax incentive. On the other hand, because the difference in tax rates between short- and long-term gains is substantial, the pressure to hold assets near the one-year mark is significant. As described above, there is evidence that taxpayers respond to the sharp line; as shown in Figure 8, taxpayers realize gains at nearly five times the rate just after the one-year line compared to the months leading up to it.109

Under a sliding scale, the tax-induced pressure to hold assets would be far less extreme in the lead-up to the one-year mark. On the other hand, a sliding scale would generate a new incentive for taxpayers to continue holding assets even after the one-year mark, until the long-

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109 See generally Dowd & McClelland, note 56.

the asset was held, ranging from 100 percent if the asset were held a year or less to only 30 percent of the gain if the asset had been held for more than 10 years.” Michael Graetz, Deborah Schenk & Anne Alstott, Federal Income Taxation: Principles and Policies 559-60 (8th ed. 2018).
term classification was fully phased in. For dates well before the one-year mark, there could be more pressure to hold the asset than under the sharp line because the probability of achieving any tax saving would increase (though the magnitude of the expected degree of tax savings might fall).

Applying our efficiency analysis to the long-term/short-term capital gain classification suggests that a sliding scale is likely to result in less deadweight loss than a sharp line. Ex ante, we have little reason to expect there to be a dip in the distribution of taxpayers’ preferred holding periods around the one-year mark. Similarly, the empirical distribution of observed holding periods also provides no evidence to support this view.

The longer the holding period over which the sliding scale is phased in, the smaller the tax incentive faced at any individual point in time to continue holding the asset. In contrast, when the sliding scale is phased in over a narrow range around the one-year mark, the behavioral effects (and deadweight loss) more closely resemble those of the sharp line. The distribution of ideal holding periods for many assets is relatively high right above zero, so one consideration in deciding where to begin the sliding scale is to avoid distorting the behavior of that group. For example, suppose the sliding scale phased in the preferential rate between six and eighteen months. Then someone whose ideal point was one month would need to hold for an additional five months to start obtaining any tax benefit—a duration we suspect is too long to distort the behavior of most investors.

Turning to classification considerations, it seems likely that a sliding scale would better further the objective of the preferential tax rate on long-term gains, but it is difficult to say for certain given that the purpose behind the provision is notoriously unclear. Commonly cited rationales include avoiding lock-in effects, offsetting the taxation of nominal gains, and rewarding investors rather than speculators. All of these objectives would seem to scale with time, consistent with a sliding scale.

Turning to complexity considerations, the additional informational complexity associated with a sliding scale would be minimal in this context. The major informational input required to determine one’s liability under this regime would still be the holding period, which, as under current law, can be determined based on the date of acquisition. No new informational inputs would be required.

With respect to computational complexity, the challenges appear manageable here as well, although there are some more difficult issues.

110 Id.
For sales of a single asset that produce a capital gain, one would simply calculate the weighted average of the long- and short-term rates, as described above. If multiple assets were sold during the year, each gain or loss could be bifurcated into a long-term and short-term component based on the sale date. So an asset sold after exactly one year that produces a $100 gain would be treated as if the taxpayer had realized $50 of long-term gains and $50 of short-term gains (assuming one year was the midpoint of the sliding scale transition range).

Overall, we conclude that switching to a sliding scale for the capital gains holding period would enhance efficiency and further the goals of the provision with very little downside in terms of additional complexity.

C. Depreciable Property Placed in Service During the Year

When taxpayers place a depreciable asset in service during the year, how much of a depreciation deduction should they be allowed? The current rule differentiates between personal property and most forms of real property. For personal property, a half-year convention is generally employed, meaning that the taxpayer treats property placed in service at any point during the year as if it had been placed in service at the midpoint of the year, entitling the taxpayer to 50% of the full year's depreciation. However, if more than 40% of the taxpayer's property is placed in service during the final quarter of the tax year, a mid-quarter convention is used instead. In contrast, the rule for real property is to apply a mid-month convention, treating the property as if it had been placed in service at the midpoint of the month. An analogous set of rules governs property that is disposed of during the tax year.

Formally, the rules for determining the allowable depreciation deduction resemble a sliding scale, in the sense that a taxpayer is entitled to a depreciation deduction in proportion to the amount of the year that transpires after the property was placed in service. In effect, however, the use of conventions converts the sliding scale into a system that more closely resembles a sharp line, or at least a hybrid of the two approaches. In particular, a half-year convention on its own would be entirely identical to a sharp line: Taxpayers who place property in service on the last day of Year 1 would be entitled to 50% of the deduction for Year 1, and 100% for Year 2. Taxpayers who place

111 IRC §168(d). The importance of the depreciation conventions for personal property is diminished by immediate expensing under §168(k), in place until 2022.
112 IRC §168(d)(3).
113 IRC §168(d)(2).
114 IRC §168(d)(4)(A).
property in service a few days later, after the start of Year 2, would not qualify for any deduction for Year 1 and would instead be allowed a 50% deduction for Year 2.

The mid-month convention retains more of the sliding scale character, but by treating the classification variable as quite coarse, it embodies some of the properties of a sharp line as well. Specifically, it creates a number of sharp lines (between each calendar month), but the importance of these lines is diminished relative to the sharp line between tax years under the half-year convention. For example, taxpayers who place property in service on January 31 would qualify for 23/24 of the deduction, whereas taxpayers who place property in service on February 1 would qualify for 21/24 of the deduction. Thus, the amount at stake is 1/12 of the deduction—far less than under the half-year convention. On the other hand, treating a single day as the level of granularity under a sliding scale would mean that a delay of one day would result in the loss of $\frac{1}{365}$ of the annual deduction.

We can apply our framework to provide some sense of the costs and benefits of various conventions. First, consider the choice between sharp lines and a pure sliding scale (by which we mean a sliding scale defined at the daily level). With respect to the efficiency properties of the two approaches, the key question concerns the timing of when taxpayers would want to place their property in service, absent tax considerations, as well as the intensity of those preferences and the likelihood that taxpayers will adjust their timing decisions based on tax considerations.

In general, a sharp line at the tax-year level (such as a half-year convention) would be preferable on efficiency grounds if taxpayers tended not to want to place additional property into service at the end of the tax year. There are a variety of factors that will influence when a firm would like to place property in service, including customer demand, availability from suppliers, seasonal fluctuations, and so forth. Overall, however, we see little reason to expect that taxpayers would be particularly reluctant to place extra property in service before the tax year’s end, so we conclude that a sliding scale is likely to be more efficient than a sharp line.

A sliding scale would probably not add much informational complexity because most businesses would already be keeping track of the date they placed property into service for accounting purposes, or, at least in moderately sized businesses, would already have records of doing so. This concern might vary across categories of property; it is probably easier to keep track of when a building was first placed into service than a small machine.
A related consideration is that under a sliding scale, all taxpayers would have to determine the precise date on which property is "placed in service" for tax law purposes. Most businesses would be able to rely on an accountant (or in the thorniest cases, a tax lawyer) to apply this definition, but this would increase costs for the firm. In addition, very small businesses might experience costs in having to undertake this legal research themselves. In contrast, under a sharp line, the only taxpayers who will need to apply this standard are those near the line, where different potential interpretations of placement in service would yield different tax years.

Finally, consider the trade-offs between a day-by-day sliding scale, and a coarser sliding scale such as the one created by the mid-month convention. From an efficiency perspective, we would typically expect the day-by-day approach to offer some advantages, since there is little reason to expect taxpayers to systematically tend not to want to place property in service toward the end of the month. On the other hand, the efficiency stakes between these options is much lower than with respect to the mid-year convention, since the tax incentive of moving from the beginning of one month to the end of the month before, while present, is much smaller in magnitude than the incentive created by the mid-year convention. At the same time, the monthly approach may offer some advantages in terms of limiting informational complexity (it is easier for businesses to keep track of the month property was placed in service than the particular day), as well as the need for taxpayers to learn and apply the details of the legal definition of "placed in service" (assuming, that is, that most reasonable interpretations would agree on the month). Because these considerations may vary by type of property, one can imagine a rationale for applying a combination of sharp lines and sliding scales of various degrees of coarseness, as the law currently does, although of course that difference in treatment can introduce its own lines (such as between personal and nonpersonal property) that create their own distortions and complexity.

D. State Residency Rules

A recurring challenge in tax law is allocating a taxpayer's income between multiple jurisdictions, such as when someone spends significant time in multiple states, countries, or even, in some cases, cities. Most jurisdictions take the approach of taxing residents

115 An additional factor exacerbating the computational complexity of a sliding scale is that there is actually legal uncertainty over the precise requirements for when an asset has been placed in service, such as whether a building that had received a certificate of occupancy and was in a condition of readiness could be considered placed in service, even though it was not yet open to the public.
differently from nonresidents. Generally, jurisdictions tax nonresidents only on income that has its “source” (i.e., it originates) there. For residents, however, jurisdictions typically tax all income, regardless of source, although typically they offer a credit for taxes paid by residents to other jurisdictions on foreign source income.

In many cases, the test for whether an individual is a resident of a jurisdiction depends at least in part on the fraction of the year the taxpayer spends there. Such tests often revolve around a sharp line. For example, people who are not domiciled in New York City but own a house or apartment there are residents of the city (and thus subject to its income tax) if they spend more than 183 days in the city limits. In a different context, European Union law has been interpreted to require that a country provide nonresidents earning 90% or more of their income in the country the same tax exemptions that are available to a resident.

Alternatively, residency classifications might turn on time spent in a jurisdiction by use of a sliding scale. That is, the degree to which a jurisdiction treats a taxpayer as a resident would vary continuously based on the fraction of the year the taxpayer spends in the jurisdiction. Thus, income sourced to a particular jurisdiction would continue to get taxed to that jurisdiction, but other income would be allocated proportionally among those jurisdictions for which a taxpayer is treated (at least to some degree) as a resident. In addition, the right to residual taxation of residents’ foreign source income might likewise be divided up proportionally.  

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116 See New York State, Income Tax Definitions, https://www.tax.ny.gov/pit/file/pit_definitions.htm. Having a New York City home and spending 184 days or more of the year in the city is sufficient, but not necessary, to be classified as a resident; someone with a domicile in New York City would also qualify.

117 To be concrete, let’s examine how these regimes might apply to a taxpayer, T, with an unclear tax domicile and an apartment in New York City. T spent 185 days in Florida and 180 in New York last year. Assume T had $400 of wages in Florida (which is Florida source income), $500 of wages in New York (which is New York source income), and $300 in dividends received during the year. Under a traditional sharp line approach, the dividends will be sourced to T’s state of residence only. For simplicity assume New York has a flat income tax of 20% on its resident’s worldwide income (and offers a credit for income taxes paid to source jurisdictions) and Florida has no income tax.

**Traditional Sharp Line:**

If T is found to be a New York resident for tax purposes, she owes New York:

$240 = 20\% \cdot (500 + 400 + 300).

T’s worldwide income is $1200 and she paid no tax to any foreign source jurisdiction, so she owes New York a full 20% on all income, or $240.

If T is a Florida resident, by contrast, she’s only taxed in New York on New York source income, which is just her New York wages, so she would owe New York:

$100 = 20\% \cdot (500)_{NY\ Source}$
There is a large literature on formulary apportionment approaches of this sort, and we will only discuss the basics of how our framework can shed light on the subject.

First, consider the efficiency trade-off. A sharp line can generate intense incentives for taxpayers to adjust where they spend their time, especially for those who are near the 183-day threshold. The efforts that taxpayers expend to avoid being characterized as residents constitutes deadweight loss to society. On the other hand, although the incentive effects from a sharp line can be large, they are limited to the relatively few taxpayers who are near the cutoff. A sliding scale, in contrast, would generate a smaller incentive for many more taxpayers. The simplest sliding scale, in which each additional day in a jurisdiction affects the taxes owed to it, would affect virtually everyone who is mobile between jurisdictions. For example, high-income taxpayers from a high-tax state would face a large incentive to spend a few extra days on vacation in a low-tax state, say Florida, to reduce their overall tax liability for the year. Similarly, tourism in high-tax states would likely suffer. Because there are so many more people who spend a small share of the year traveling between states for vacation or work than there are who spend near-equal amounts of the year traveling between states, a sharp line is probably more efficient than this pure form of a sliding scale. Put differently, there likely is a large dip in the distribution of ideal points in this case.

On the other hand, a sliding scale with a de minimis exception could well be more efficient than either a pure sliding scale or a sharp line. Under this approach, taxpayers would be treated as part-year residents

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**Sliding Scale Only for Apportioning Income Whose Source Follows T’s Residence:**

Suppose a sliding scale is adopted for apportioning dividends and other income whose source follows the taxpayer’s residency. In this case, the $300 of dividends would be proportionally split: $300 \cdot \frac{180}{365} \text{ as New York source and } $300 \cdot \frac{185}{365} \text{ as Florida source.}

Under this system, if \( T \) is a New York resident, she still owes New York $240. This is because, as above, if \( T \) is a New York resident, the sourcing between New York and Florida is irrelevant because she paid no tax to Florida. She has $1200 of total income and no tax credits, so she owes $240. If \( T \) is a Florida resident, she owes New York:

\[
$129.58 = 20\% \cdot \left( $500 + $300 \cdot \frac{180}{365} \text{ as NY Source} \right)
\]

**Full Sliding Scale:**

By contrast, a full sliding scale would implicitly also proportionally divide up the right of the taxpayer’s state of residence to residually tax foreign source income.

Mechanically the sliding scale tax liability would just be a weighted average of how much the taxpayer would owe if she were a resident and how much she would owe if not, multiplied by the days in the state. So the taxpayer would owe New York:

\[
$169.04 = \left( $240=\text{tax if NY Resident} \cdot \frac{180}{365} \right) + \left( $100=\text{tax if Not NY Resident} \cdot \frac{185}{365} \right)
\]

and the domicile determination would be irrelevant.
of any state in which they spent more than, say, 90 or 120 days during the tax year. Income would then be allocated among the states in which the individual was at least a part-year resident, in proportion to the days above the de minimis threshold spent in the state. Depending on the distribution of taxpayers' ideal points above the de minimis threshold, this modified sliding scale could well be more efficient than the sharp line: Its distortionary effect would be smaller for the taxpayers affected by the sharp line, and its effects would still be limited to a relatively small number of taxpayers whose days in another state exceeded or could plausibly be altered to exceed the de minimis threshold.

Informational complexity is another important concern with a sliding scale approach to state residency determinations. It could be quite burdensome for taxpayers to keep track of the precise number of days they spend in any particular state. Of course, taxpayers near the line already have to keep track of this information, but that requirement affects a small share of taxpayers. Again, however, a modified version of the sliding scale could alleviate some of the complexity costs, so that the only taxpayers who would need to keep track of their dates would be the relatively small group who spend significant fractions of the year in multiple jurisdictions. In addition, it is possible that new technology, such as location-tracking capabilities on one's phone, could ease the record-keeping costs for tracking location over the course of the year.

With respect to tax planning, a sliding scale could actually reduce the computational challenges associated with tax planning about residency decisions, since taxpayers would not need to spend as much effort predicting the precise number of days in the year that they will be in the jurisdiction. That is, under a sharp line, the costs of spending an extra day in a jurisdiction hinge on the likelihood and importance of spending future days in that jurisdiction during the tax year. Under a sliding scale, in contrast, the tax implications of spending time in the jurisdiction do not vary in this way.

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118 Under this proposal, taxpayers who are not residents of any one state for at least ninety days could have their incomes allocated across states using a smaller de minimis threshold (e.g., sixty days).

119 If there is a fixed (e.g., computational) cost to participating in each jurisdiction's tax regime, a system like this could generate additional deadweight loss by inducing taxpayers to bunch just below the number of days at which they must begin computing tax liability under the second jurisdiction's regime.

120 A different consideration is that to the extent jurisdictions require different information because they use different definitions of the tax base this will also add to informational complexity.
E. Debt-Equity Divide for C Corporations

Economically, debt and equity are both claims on the cash flows of business organizations that share in the risk and return of the enterprise.\textsuperscript{121} Debt and equity, therefore, can be thought of as ends of a spectrum whose underlying variable is a measure of the risk borne by the holder and the expected return to the instrument.

Tax law nevertheless utilizes a sharp line that classifies nearly all capital instruments as either entirely debt or entirely equity. To see the sharp line, start by imagining paradigmatic debt: an unsubordinated, unconditional promise to pay a sum certain with fixed interest at a fixed date, issued by a well-capitalized firm.\textsuperscript{122} Now start adding equity-like features. Subordinate the instrument to existing debt holders; make the periodic “interest” payments vary directly with the returns of the company; add a provision allowing conversion of the instrument into common stock. At some point—the law is very hazy on where—you will have turned your instrument into equity, and after that, adding further equity-like features will have no effect on the classification. This is the sharp line.

Would a sliding scale work better here? Congress apparently has at least toyed with this idea. In 1989, it revised § 385 to explicitly permit Treasury to promulgate regulations that would treat issuances that fall in the middle of the spectrum as part debt and part equity.\textsuperscript{123} Courts have occasionally also bifurcated instruments.\textsuperscript{124} Moreover, there is anecdotal evidence of firms trying to create instruments that are as equity-like as possible while still trying to qualify for the (usually) more favorable debt treatment\textsuperscript{125} and a sense this problem is increasing.\textsuperscript{126}

\textsuperscript{121} Richard Brealey, Stewart Myers & Franklin Allen, Principles of Corporate Finance 230-35 (9th ed. 2007). Any debt instrument can be represented as a risk-free bond and a short position in put options on the company’s common stock. Because all debt creates an implicit short position in the company’s put options, this means that even paradigmatic debt should lose a bit of value when the company’s equity falls. This in turn reflects the fact that every debt instrument—in the worst cases—may become the residual claimant on the company’s cash flows (if things go badly enough), the position we usually associate with equity. Thus, no debt instrument is entirely free of the risks of the enterprise.

\textsuperscript{122} See IRC § 385.


\textsuperscript{124} See, e.g., Fin Hay Realty Co. v. United States, 398 F.2d 694, 697 (3d Cir. 1968).

\textsuperscript{125} See Polito, note 123, at 789 (discussing adjustable rate convertible loans (ARCNs), which were designed to meet the safe-harbor entitled an instrument to debt treatment under Treasury’s proposed regulations under IRC § 385, while being as far to the equity side of the spectrum as possible).

\textsuperscript{126} Wolfgang Schön, The Distinct Equity of the Debt-Equity Distinction, Bull. for Int’l Tax’n, Sept. 2012, at 490, 491, https://ssrn.com/abstract=2444648 ("In former times, the rights of a shareholder and the rights of a bondholder could easily be distinguished from each other, but modern financial engineering has created an infinite continuum of financial...\)
Nevertheless, after looking at the data, we are skeptical that creating a sliding scale would yield large efficiency gains, at least for public C corporations in their domestic operations. In Figure 11 we present a very rough cut of the capital structure of U.S. public companies:

**FIGURE 11**
**AGGREGATE CAPITAL STRUCTURE OF U.S. PUBLIC CORPORATIONS**

![Bar chart showing capital structure](image)

Source: Authors' calculation from Compustat data consolidated at the parent company level, as of April 2018.

From an efficiency point of view, these data do not provide much support for adopting a sliding scale here.\(^{127}\) The data do not show a large bulge of capital issuances just on the tax-favored side of the debt-instruments where more or less any combination of financial elements (risks and rewards) and other entitlements (voting rights, cancellation rights, conversion rights, etc.) can be put together.\(^{126}\).

\(^{127}\) It is worth noting some caveats here. First, the data only cover public companies. Private companies using venture capital likely make much greater use of instruments that are in the middle of the spectrum—for example, convertible preferred stock. That said, public corporations currently make up the vast majority of the tax base of C corporations, so our analysis should be valid for C corporations. Second, our data are consolidated at the level of the parent company. Our analysis does not speak to the capital structure within U.S. multinationals and whether companies may be manipulating the debt-equity divide to engage in profit shifting or otherwise avoid taxes in the international realm. See Schön, note 126. Finally, the consequences of being on one side or the other of the debt-equity divide depend on, among other things, the corporate tax rate, the dividend and/or capital gains rate, the (top) tax rate on ordinary income, and any limitations on the deductibility of interest. At the moment, those factors combine to make the differences between debt and equity much smaller for C corporations than under prior rate structures.
equity line, unlike with, say, capital gains realization. Instead, the issuances seem to be quite bimodal, with the vast majority of issuances occurring toward one pole or the other of the debt-equity spectrum. This in turn suggests that implementing a sliding scale might well be less efficient than the existing sharp line, even if there were no complexity and administrative costs to doing so. Facing a sliding scale, firms that would, absent taxes, issue equity may slide down toward debt a bit, even if they wouldn’t have found it profitable to slide all the way past the old sharp line.\textsuperscript{128}

Complexity and administration would also pose serious challenges to converting to a sliding scale. Congress and Treasury have struggled without much success to give more clarity to the substantive law on what distinguishes debt and equity.\textsuperscript{129} Implementing a sliding scale would require substantially more guidance for placing instruments along the debt-equity spectrum.

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{128} An example may illustrate the strengths and weaknesses of the analysis above. Our model assumes that firms face convex costs in moving away from their ideal point. Assume a company, \textit{ABC}, would, absent taxes, raise additional capital as pure equity. Convex costs imply that if taxes induce \textit{ABC} to issue debt instead, it will choose floating rate notes rather than going even farther from its ideal point and issuing fixed-rate bonds. As a result, if our model is right, we can assume that the relatively small number of capital issuances just on the debt side of the threshold indicates that taxes are not distorting the issuances of \textit{ABC} and similarly situated firms very often.

There is an important caveat to the analysis in the preceding paragraph, however. Tax law's way of breaking down equity and debt may not reflect sound economics. Thus, a thirty-year bond that pays a fixed rate guarantees \textit{ABC}'s access to capital for a long time. This bond is arguably a closer economic substitute for pure equity-like common stock—which gives \textit{ABC} access to capital indefinitely—than a two-year floating rate note whose rate depends on the performance of \textit{ABC}. Tax law nevertheless considers the two-year floating rate note closer to equity than the thirty-year bond. As a result, if taxes distort \textit{ABC}'s decision, it may well choose a thirty-year bond and not the floating rate notes, both of which are treated as debt for tax purposes, but the thirty-year bond is economically closer to \textit{ABC}'s ideal point (even if tax law doesn't see it that way). Even if this is true, Figure 11 still indicates that it would not increase efficiency much to create a sliding scale based on tax law's existing definitions of debt and equity.

If such a sliding scale were adopted, the two-year floating rate notes would be taxed as part equity and part debt, while the thirty-year fixed rate bonds would be taxed as all debt. Under such a system, \textit{ABC} would still issue the thirty-year bonds. So, no distortion is eliminated for firms like \textit{ABC} by the sliding scale. Meanwhile, some firms currently issuing equity at their ideal points under the sharp line might switch under the sliding scale to preferred stock, which would now be taxed partly like debt, just to get the tax advantages.

\item \textsuperscript{129} The House's proposed version of the Internal Revenue Code of 1954 would have provided a more rulelike approach to debt versus equity, but the Senate Finance Committee deleted the proposed language, finding that "precise definitions which will classify for tax purposes the many types of corporate stocks and securities will be frustrated by the numerous characteristics of an interchangeable nature which can be given to these instruments." S. Rep. No. 83-1622, at 4763 (1954). Likewise, when Congress added IRC § 385 in 1969, it called on Treasury to promulgate regulations to more clearly define the debt-equity distinction. Treasury proposed regulations in 1980, but they were criticized and subsequently withdrawn before going into effect. See also Polito, note 123, at 783-90.
\end{itemize}
\end{footnotesize}
One could imagine a checklist approach using the primary measures seen in the statute and case law—say five of them—like: Is there a fixed repayment date with fixed interest? Is the instrument subordinated to other capital providers, etc.? Falling on the debt side of all five factors would create 100% debt; being on the debt side of four of the factors would result in an instrument treated as 80% debt and 20% equity and so on. The downside of such a system is that it may be easily manipulated by taxpayers and would require evaluation of each of the five factors by both taxpayers and the IRS for each issuance.

A more ambitious approach might equate equity with the amount of risk borne by the holder and reflected in the expected return of the issuance. A security would then be placed on the debt-equity spectrum relative to the lowest-expected return security issued by the corporation that is assumed to be pure debt. This approach would be less easily manipulated than the checklist approach but would likely be more difficult for both taxpayers and the IRS to apply.

IX. CONCLUSION

By studying the behavioral effects of sliding scales and sharp lines under a range of assumptions, we have identified factors that tend to make sliding scales more efficient than sharp lines for raising revenue—at least when the distribution of taxpayer preferences does not make it possible to draw the sharp line in a non-distorting way. Thus, we think it makes sense for policymakers to start from the presumption that a sliding scale would be more efficient. In some cases, there might be good reasons to suspect the distribution of taxpayer preferences is such that a sharp line would be more efficient, thus overturning the presumption. And in other cases, a sliding scale will be much more complex than a sharp line, undermining the case for its adoption. Ultimately, the determination as to which type of instrument is best should be made setting by setting, but we hope the general principles outlined here can help guide that analysis.

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For a proposal along these lines, see Polito, note 123, at 791-810. Note that this approach partially accords with the existing case law, which, through multiple factors, seeks in part to determine to what extent the holder of the issuance bears the risk of the enterprise. Note, however, that the expected return of an issuance corresponds imperfectly to the risks of the enterprise as examined in the case law because most risks specific to the firm can be diversified away at low cost and will thus not be compensated in markets or affect the expected return. See, e.g., Ayres & Fox, note 40, at 453-54.
APPENDIX

EFFICIENCY ADVANTAGE OF SLIDING SCALE WITH UNIFORM IDEAL POINT DISTRIBUTION

Consider a population of taxpayers who select values of \( x \in [0,1] \) and pay some tax that depends on their choice of \( x \). Individuals choose \( x \) to maximize

\[ u = -\alpha(x - x^*)^2 - T(x) \]

where \( x^* \) denotes the individual’s ideal point, \( \alpha > 0 \), and \( T \) denotes the function that maps behavior into tax liability, \( T: [0,1] \to \mathbb{R} \). Individuals are heterogeneous only in their ideal points, so we can write individual behavior as \( x = x(x^*, T) \). The distribution of ideal points in the population is denoted by \( f \). Social welfare under some tax function \( T \) is therefore given by

\[ w(T) = \int_0^1 \left( -\alpha(x(x^*, T) - x^*)^2 - T(x(x^*, T)) \right) f(x^*)\,dx^* \]

The tax function can either take the form of a sliding scale, \( T_{SS}(x) = \tau x \), or a sharp line with cutoff \( \frac{1}{2} \):

\[ T_{SL}(x) = \begin{cases} 
0 & \text{if } x \leq \frac{1}{2} \\
\tau & \text{if } x > \frac{1}{2} 
\end{cases} \]

The government’s objective is to maximize social welfare subject to a revenue constraint \( \overline{R} \):

\[ \max_{T \in \{T_{SS}, T_{SL}\}} w(T) \quad \text{s.t.} \quad \int_0^1 T(x(x^*, T)) f(x^*)\,dx^* \geq \overline{R} \]

**Proposition**

Suppose that \( f \) is uniform. Let \( T_{SL} \) denote a sharp line tax schedule that satisfies the revenue constraint. Then there exists a sliding scale tax schedule, \( T_{SS} \), such that \( T_{SS} \) satisfies the revenue constraint and yields higher social welfare, \( w(T_{SS}) > w(T_{SL}) \).

**Proof**

Suppose that \( T_{SL} \) satisfies the revenue constraint, and let \( \tau \) denote the amount of tax that is owed under \( T_{SL} \) when \( x \) exceeds the cutoff.
Taxpayers choose $x$ by comparing their utility at $x = \frac{1}{2}$ to the utility achieved at their ideal point, which yields the following decision rule:

$$x(x^*, T^{SL}) = \begin{cases} 
  x^* & \text{if } x^* \leq \frac{1}{2} \\
  \frac{1}{2} & \text{if } \frac{1}{2} < x^* \leq \frac{1}{2} + \frac{1}{\sqrt{\alpha}} \\
  x^* & \text{if } x^* > \frac{1}{2} + \frac{1}{\sqrt{\alpha}}
\end{cases}$$

(1)

The revenue raised under $T^{SL}$ is given by $r(T^{SL}) = \int_{\frac{1}{2} + \frac{1}{\sqrt{\alpha}}}^{1} \tau f(x^*)dx^* = \tau \left(1 - F\left(\frac{1}{2} + \frac{1}{\sqrt{\alpha}}\right)\right)$, where $F(\cdot)$ is the population ideal type cdf. Because $f$ is uniform over $[0,1]$, we have $r(T^{SL}) = \tau \left(\frac{1}{2} - \frac{1}{\sqrt{\alpha}}\right)$. Without loss of generality, we assume that $\tau$ is on the correct side of the Laffer curve, so that $\frac{\partial r}{\partial \tau} = \frac{1}{2} - \frac{3}{2}\sqrt{\alpha} \geq 0$, or

$$\tau \leq \frac{\alpha}{9}$$

(2)

Now consider a sliding scale tax schedule for which the tax rate is set to the level of the tax under the sharp line, $T^{SS}(x) = \tau x$. Our strategy will be to show that this new tax raises at least as much revenue as $T^{SL}$ and also achieves higher social welfare.

Under $T^{SS}$, taxpayers select $x$ to solve: $\max_{x \in [0,1]} -\alpha(x - x^*)^2 - \tau x$, which yields the decision rule:

$$x(x^*, T^{SS}) = \begin{cases} 
  x^* - \frac{\tau}{2\alpha} & \text{if } x^* \geq \frac{\tau}{2\alpha} \\
  0 & \text{if } x^* < \frac{\tau}{2\alpha}
\end{cases}$$

(3)

Total revenue under the sliding scale is therefore given by $r(T^{SS}) = \int_{\frac{\tau}{2\alpha}}^{1} \tau \left(x^* - \frac{\tau}{2\alpha}\right)f(x^*)dx^* = \int_{\frac{\tau}{2\alpha}}^{1} \tau \left(x^* - \frac{\tau}{2\alpha}\right)dx^*$, where the second equality follows from the assumption that $f$ is uniform. Adding and subtracting $\int_{\frac{\tau}{2\alpha}}^{1} \tau \left(x^* - \frac{\tau}{2\alpha}\right)dx^*$ allows us to write $r(T^{SS}) = \int_{0}^{\frac{\tau}{2\alpha}} \tau \left(x^* - \frac{\tau}{2\alpha}\right)dx^*$, where the second term is negative because $(x^* - \frac{\tau}{2\alpha}) \leq 0$ for $x^* \in [0, \frac{\tau}{2\alpha}]$. Hence, $r(T^{SS}) \geq \int_{0}^{\frac{\tau}{2\alpha}} \tau \left(x^* - \frac{\tau}{2\alpha}\right)dx^*$. Note that the second term is negative because $(x^* - \frac{\tau}{2\alpha}) \leq 0$ for $x^* \in [0, \frac{\tau}{2\alpha}]$. Hence, $r(T^{SS}) \geq \int_{0}^{\frac{\tau}{2\alpha}} \tau \left(x^* - \frac{\tau}{2\alpha}\right)dx^*$.
\( \frac{r}{2a} \) \( dx^* = \frac{\tau}{2} - \frac{\tau^2}{2a} \). Comparing this expression to \( r(T^SL) \) yields a sufficient condition under which the sliding scale generates more revenue than the sharp line: \( r(T^SS) > r(T^SL) \iff \tau < 4\alpha \). Because \( T^SS \) is on the correct side of the Laffer curve, we have from (2) that \( \tau \leq \frac{\alpha}{9} < 4\alpha \), and hence, \( r(T^SS) > r(T^SL) \). Therefore, because \( T^SL \) satisfied the revenue constraint, it must therefore also be the case that \( T^SS \) satisfies the revenue constraint.

Next, we will show that \( w(T^SL) < w(T^SS) \). By definition, \( w(T^SL) = \int_0^1 -\alpha (x(x^*, T^SL) - x^*)^2 f(x^*) dx^* = w(T^SL) = \int_0^1 -\alpha (x(x^*, T^SL) - x^*)^2 dx^* \), where the second equality follows from the uniform distribution of \( x^* \). Using (1), we have \( w(T^SL) = -\alpha \int_{\frac{1}{2}}^{\frac{1}{2} + \sqrt{\tau/\alpha}} \left( \frac{1}{2} - x^* \right)^2 dx^* \). Evaluating this integral yields \( w(T^SL) = -\frac{\tau^{3/2}}{3\alpha^{1/2}} \).

Turning to welfare under the sliding scale, following the same steps as before and using (3), we can write \( w(T^SS) = \int_0^{\tau/2a} -\alpha (x^*)^2 dx^* + \int_{\tau/2a}^{\tau} -\alpha \left( \frac{\tau}{2a} \right)^2 dx^* \). Adding and subtracting \( \int_0^{\tau/2a} \alpha \left( \frac{\tau}{2a} \right)^2 dx^* \) yields \( w(T^SS) = -\int_0^{\tau/2a} \alpha \left( \frac{\tau}{2a} \right)^2 dx^* + \alpha \int_{\tau/2a}^{\tau} \left( \frac{\tau}{2a} \right)^2 - (x^*)^2 \) \( dx^* \geq -\int_0^{\tau/2a} \alpha \left( \frac{\tau}{2a} \right)^2 dx^* = -\frac{\tau^2}{4\alpha} \), where the inequality follows from the fact that \( \int_{\tau/2a}^{\tau} \left( \frac{\tau}{2a} \right)^2 - (x^*)^2 \) \( dx^* \) is weakly positive (given the limits of integration). Combining these results, a sufficient condition for \( w(T^SS) > w(T^SL) \) is \( -\frac{\tau^2}{4\alpha} > \frac{-\tau^{3/2}}{3\sqrt{\alpha}} \iff \tau < \frac{16}{9} \alpha \), which is implied by (2).