1998

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LOGIC AND ELEMENTS

Richard D. Friedman*

We may happily agree with Holmes that logic is not the life of the law\(^1\) and yet contend that logic should play a significant role in legal discourse. Logic cannot demonstrate the truth of premises, and so by itself it cannot demonstrate the merits of a legal argument. Moreover, even given the premises, it may be that a leap of faith, or intuition, has an irreducible role at least in some good legal arguments.\(^2\) But at least a sound legal argument will not be an illogical one. An argument will not be persuasive if it appears to violate basic principles of logic. If the enunciation of a principle of law is to have any hope of stability it must be capable of consistent application in situations that are materially similar, and if the articulation is not logically coherent this condition is unlikely to prevail.

In this paper, I will explore the use of symbolic logic in discussing the notion of elements of a claim or a defense. I will use the term "claim" in a general sense, referring not only to the grounds underlying an action—what is asserted in a complaint or charging instrument—but also to any demand for judicial relief. Thus, I will speak of a claim for preliminary relief or for discovery sanctions.

In Part I, I will suggest that the attempt to state a legal argument by setting out the elements of the claim or the defense symbolically can often assist the quest for clarity. In Part II, I will explore some of the limits of this technique, showing that in some cases the very notion of the elements of a claim or a defense breaks down. In Part III, I will suggest the outlines of a logic for legal argument that operates defeasibly and so is significantly different from the classical logic

\* Professor of Law, University of Michigan Law School. Many thanks to Howard Pospesel, Bob Rodes, Kevin Saunders, and Doug Walton, especially for reducing my level of ignorance about non-monotonic reasoning, but also for many other helpful, and encouraging, suggestions and comments.

1 See Oliver Wendell Holmes, Jr., The Common Law 1 (Boston, Little, Brown & Co. 1881).

presented vividly and engagingly in a wonderful new text, Premises and Conclusions, by Robert Rodes and Howard Pospesel.3

I. PAYING ATTENTION TO THE STRUCTURE OF A CLAIM

When I judge moot court arguments, I often ask a student a question such as, “Well, suppose we disagree with you on Point I of your argument. Do you lose?” or perhaps, “OK. Suppose we agree with you on Point I. Do you win?” And sometimes I am disappointed to realize that the student cannot answer with confidence—which usually means that he or she has not thought out the structure of the argument with sufficient care. I do not believe it is necessary that most lawyers become comfortable, or even understand, the full rigor and richness of Rodes and Pospesel’s exposition. But I do believe that legal discourse is hindered substantially if lawyers do not understand or are unable to present the structure of a claim with a substantial degree of rigor. (Note the disjunctive in that sentence, by the way. Perhaps the sentence has a superfluous clause in that “unable to present” implies “do not understand.” But even if this is so—I am not sure that it is—the hindrance is probably greater when “unable to present” as well as “do not understand” is true.)

I find that symbolic logic often helps in achieving the requisite degree of rigor. In this Part, I will illustrate how. For now, I will use the same notation as do Rodes and Pospesel.4 Some of what follows is not only straightforward but also so simple that perhaps I should be embarrassed walking through it in the shadow of the highly sophisticated analysis of Rodes and Pospesel. But, as I have suggested, I believe many capable students, and presumably lawyers as well, fail to engage in the type of elementary logical analysis illustrated here, and so fail to understand fully the logical structure and consequences of their cases. Moreover, the elementary uses of logic provide a platform on which to analyze more complex problems and to consider more sophisticated concepts.

A. Basic Structures

Consider first a claim with a simple conjunctive structure. Suppose the structure is

4 Thus, A → B means “A implies B,” A ↔ B means “A is equivalent to B,” A ∨ B means “A or B,” A & B means “A and B,” (x) means “For all x it is true that . . . ,” and ∃x means “There exists an x such that . . . .”
(A & B & C) ↔ L,  

where A, B, and C are each disputed propositions and L is the proposition that liability is established. In this case, it should be clear that the plaintiff (assuming that is the party arguing for liability) wins if and only if each of the propositions A, B, and C is true. And because it follows from Proposition 1 that

\[-(A \& B \& C) \leftrightarrow \neg L\]  

and

\[-(A \& B \& C) \leftrightarrow \neg A \vee \neg B \vee \neg C,\]  

it is enough for the defendant to win that the court be persuaded that any one of the propositions A, B, or C is false.

Now consider a claim with a simple disjunctive structure. Suppose the structure is

\[(A \vee B \vee C) \leftrightarrow L.\]  

It is elementary that in this case the plaintiff wins if any one or more of the three propositions, A, B, and C are true. And because it follows from Proposition 4 that

\[-(A \vee B \vee C) \leftrightarrow \neg L,\]  

and

\[-(A \vee B \vee C) \leftrightarrow \neg A \& \neg B \& \neg C,\]  

the defendant can win if and only if each of the three propositions A, B, and C appears to be false.

B. Applications for Issue Preclusion

Setting out the elements of a claim symbolically is often particularly helpful in analyzing matters of issue preclusion. The general verdicts issued by many American civil juries often leave an open question of whether a particular issue was actually decided in the first litigation. Logical analysis can sometimes resolve the problem.  

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5 I am putting aside for now the question of the burdens of proof. I will address the burden of pleading later. I am also putting aside the so-called problem of conjunction. This is, the question of what happens if the factfinder is persuaded that each of the elements of the claim is established to the requisite level of confidence, but that the aggregated proposition—here (A & B & C)—is not. I address this problem at some length in Richard D. Friedman, Answering the Bayesioskeptical Challenge, 1 INT'L J. EVIDENCE & PROOF 276, 279–84 (1997).

6 Kevin Saunders, a participant in this symposium, has presented a far more sophisticated logic for evaluating matters of issue preclusion in A Logic for the Analysis of Collateral Estoppel, 12 RUTGERS COMPUTER & TECH. L.J. 99 (1986). For most workday
purposes, however, I believe the analysis presented here functions just as well, and without the disadvantage of a considerable degree of extra complexity.

Saunders focuses on the problem that, because factfinding in litigation is probabilistic—that is, a factfinder may conclude for purposes of the litigation that a given proposition is true even in the face of uncertainty as to its truth—prior findings that each of two or more propositions is true do not necessarily imply that the factfinder would conclude that the conjunction of the propositions is true. This is an interesting variation on the much discussed “problem of conjunction,” to which I have referred, see supra note 5. Saunders’ analysis operates on the premise, with which I agree, that if (1) P and Q are the essential elements of the plaintiff’s claim, (2) the governing standard of persuasion is “more likely than not,” and (3) the factfinder concludes that P and Q each satisfy this standard but that the conjunction P & Q does not, then the defendant is entitled to judgment.

I do not believe that the problem as posed by Saunders is usually a serious difficulty, however: If P and Q were both found in litigation previous to one in which the plaintiff seeks to have the conjunction P & Q established preclusively, it will usually be because both P and Q were established in one prior litigation, and the conjunction P & Q was a necessary finding for the outcome in that litigation.

Interestingly, Saunders’ analysis seems to point to a broader problem that neither it nor prevailing preclusion doctrine solves. Suppose that in prior litigation, under circumstances satisfying the preconditions for preclusion, a jury must have found P against the current defendant, and that in the current litigation the plaintiff needs to establish the conjunction P & Q to win the action. The defendant may argue,

True, the prior jury found P against me, but it was not necessarily certain on that point; presumably the jury had at least some uncertainty. If the current jury were trying P and it had some uncertainty about that proposition, that uncertainty would raise the probability to which it would have to find Q in order for it to find the conjunction P & Q against me. I’m entitled to the benefit of that uncertainty! You mustn’t let it evaporate by instructing the current jury to treat P as established to a certainty when there is no basis for concluding that this is what the prior jury found.

Note that if the court tries to appease the defendant by instructing the current jury to treat P as established only to the minimum level of confidence that the prior jury might have had, this would undercut preclusion doctrine very substantially. A finding that P was barely more likely than not presumably could have supported the judgment in the prior action, if all other elements of the claim in that action were established with near certainty. If the current jurors were instructed to treat P as established only to the extent of being “barely more likely that not” that would mean that they could not find for the plaintiff unless they found Q nearly to a certainty.

I believe this is a quite difficult problem. In Answering the Bayesioskeptical Challenge, Friedman, supra note 5, at 284 nn.17–18, I analyzed the related problem of how, if the trial is bifurcated, with a trial first of P and then of Q, the factfinder’s uncertainty at the first stage might be imported into the second stage. A suggestion made there—that the factfinder at the first stage be asked to make some (presumably non-numerical) assessment of its uncertainty—might be appropriate in the preclusion context as well, if the possibility of future preclusion is anticipated at the first trial. In a bench trial, this should be relatively easy given the court’s articulation of its reasoning, and even if future preclusion is not anticipated at the first trial, the second judge
sider first Variation 1 of a classic hypothetical. Alfred sues Barbara for auto negligence in a state that has a contributory negligence defense, but no compulsory counterclaim rule. Barbara raises the contributory negligence defense and wins a general verdict. Then Barbara sues Alfred, and Alfred raises the contributory negligence defense. What effect does the doctrine of issue preclusion have? To win in Action 1, Alfred would have had to prevail on the proposition (¬A & B), where A and B represent, respectively, the negligence of Alfred and Barbara. Barbara’s verdict means that the jury found (A v ¬B), but we cannot know for sure whether the jury found A or whether it found ¬B—and a fortiori we cannot know whether the jury found (A & ¬B), which is what Barbara would have to demonstrate to prevail in her action. There is no preclusive effect.

Now consider an actual textbook case, Illinois Central Gulf Railroad v. Parks, in which Bertha Parks won a negligence action against the railroad. Thus, she must have persuaded the first jury of (Nr & ¬Nb & Ib), where Nx is the negligence of x, Ix is personal injury to x, r is the railroad, and b is Bertha. In the same action, her husband Jessie, who sought damages for loss of Bertha’s services and consortium, lost a general verdict. Contributory negligence would defeat Jessie’s claim. It appears, therefore, that the jury must have been persuaded of (¬Nr & ¬Nj & Cj), where j represents Jessie and Cx represents the damages to x for loss of consortium. Jessie then brought a separate action for his personal injuries. What preclusive effect does the first action have? To prevail in the second action, Jesse must prove (Nr & ¬Nj & Ij). Nr was essential to the judgment against the railroad in Bertha’s action, and so the railroad may be precluded on that issue. But is Jessie precluded with respect to Nj? The verdict in the first action against Jessie means that the jury found (¬Nr v Nj v ¬Cj). We know from the verdict for Bertha that the jury did not find ¬Nr. The railroad, trying to win by a process of elimination, further contended that

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might be able to discern the level of uncertainty of the first judge (who might be the same person). In a jury trial, a special interrogatory would be necessary on the verdict form ("Which best expresses your level of confidence in P? Virtually certain; highly likely; substantially more likely than not; barely more likely than not.").

8 390 N.E.2d 1078 (Ind. Ct. App. 1979), excerpted in Yeazell, supra note 7, at 819.
9 Some courts presumably would have held this action barred by claim preclusion, on the ground that Jessie should have asserted the claim in the first action; apparently the governing law of claim preclusion was not so aggressive, however.
10 There is a non-mutuality issue—given that Jessie could not have been precluded by a finding in favor of the railroad in Bertha’s action against it—but many courts, like the Parks court, would not be deterred by this factor from ruling in favor of preclusion.
the jury could not have found \(-C_j\) because Jessie’s evidence on this point was uncontroverted. But the court, perhaps overly generously, accepted Jessie’s argument that his evidence on consortium in the first action was minimal and may have been rejected by the jury. So Jessie got the sweet of issue preclusion without the bitter.

Now turn to another type of problem—when we know just what the factfinder decided, but at least arguably it decided more than was necessary for resolution of the case. Consider Variation 2 of the Alfred-Barbara hypothetical.\(^{11}\) This is the same as Variation 1 except that the jury in Action 1 specifically finds \((A \& B)\)—that is, both parties are negligent, leading to a judgment for Barbara. If we take the view of the Restatement (Second) of Judgments,\(^{12}\) that an issue is precluded only if it was “essential to the judgment,” what is the preclusive effect now? We can ask whether a finding was essential to the judgment by asking whether the judgment would stand absent that finding. B was not essential to the judgment, because a finding of \((A \& -B)\) would, \textit{a fortiori}, lead to judgment for Barbara. But A was essential to the judgment for Barbara, because a finding of \((-A \& B)\) would have led to a judgment for Alfred.

With respect to this type of problem, too, symbolic logic can help analyze actual cases as well as hypotheticals. In \textit{Halpern v. Schwartz},\(^{13}\) any one of three determinations would have been sufficient (given other requirements) to drive Halpern into bankruptcy. The trial court made all three—\(H \& C \& P\), where \(H\) is a transfer with intent to hinder and delay creditors, \(C\) is a transfer without fair consideration, and \(P\) is preferential payment of a prior debt. The United States Court of Appeals for the Second Circuit affirmed, but without opinion.\(^{14}\) In Action 2, Halpern sought a discharge from bankruptcy, which would be denied if \(H\) were established; \(C\) and \(P\) did not bear on the discharge question. This time, the Second Circuit held that Halpern was not precluded from challenging \(H\) by the finding against her in Action 1. The court believed that if the trial court in Action 1 had based its decision on multiple grounds, it was less likely to give each one serious consideration and that vigorous review of each one would be unlikely.\(^{15}\)

Now compare \textit{Winters v. Lavine},\(^{16}\) decided by the same court. Winters sought Medicaid compensation in New York courts for the

\(^{11}\) See YEAZELL, \textit{supra} note 7, at 831 note 6.
\(^{12}\) \textit{Restatement (Second) of Judgments} \S 27 (1982).
\(^{13}\) 426 F.2d 102 (2d Cir. 1970), \textit{excerpted in YEAZELL, \textit{supra} note 7, at 824}.
\(^{14}\) See \textit{id}. at 103.
\(^{15}\) See \textit{id}. at 105.
\(^{16}\) 574 F.2d 46 (2d Cir. 1978), \textit{excerpted in YEAZELL, \textit{supra} note 7, at 827}.
services of a Christian Science practitioner. She was defeated on two grounds, that the statute constitutionally denied her the claimed benefits and that her proof of illness and treatment was inadequate. She then brought a federal civil rights action challenging the denial. The Second Circuit held that she was precluded by the results of the first action. The court essentially rejected the analysis of Halpern, limiting its effect to the bankruptcy context and approving the stance of the first Restatement that "where the judgment is based upon the matters litigated as alternative grounds, the judgment is determinative of both grounds, although either alone would have been sufficient to support the judgment."

It seems to me, however, that the results—though not the underlying theory—in Halpern and Winters might be broadly reconcilable. The holding of Halpern is essentially that, although the trial court in Action 1 determined (H & C & P), the first judgment could only be preclusive of (H v C v P), because this is the minimum we know was necessary to support the result of Action 1. Applied to Winters, this would mean that the preclusive effect of Action 1 would be limited to (C v P)—that is, that either the statute constitutionally denied Winters the claimed benefits or that her proof was inadequate. But now notice the different posture of the respective second actions. In Halpern, only H mattered in Action 2, and (H v C v P) does not imply H. In Winters, though, it appears at least plausible that (C v P) would defeat the plaintiff in Action 2. That is, if in Action 2, the court accepts that Action 1 established at least the proposition that one basis or the other for the result in that action should be deemed to be true, and if either one would be sufficient to defeat Winters in Action 2, the court in Action 2 might conclude that she should be precluded on the basis of the disjunctive proposition, without establishing one side or the other of the disjunction. This argument might be fortified by a point that I will now argue, that all or practically all claims in fact have disjunctive as well as conjunctive aspects.

17 See id. at 51.
19 Restatement of Judgments § 68, cmt. n (1942), quoted in Winters, 574 F.2d at 67.
II. Complicating, and Breaking Down, the Structure of a Claim

In Part I, I have tried to show how thinking about the structure of a claim in terms of its elements can be useful, and even essential, in some circumstances. But now I will discuss some complications in the nature of a claim—complications that sometimes, I believe, break down the elemental approach.

In Part I, I have discussed claims that had either conjunctive or disjunctive aspects, but not both. It is apparent that a claim will not always have so simple a structure. As rendered by Rodes and Pospesel, 20 42 U.S.C. § 402, part of the Social Security Act, entitles a person to an old-age insurance benefit under the following condition, which for present purposes we may assume is necessary as well as sufficient:

\[ I_x \land A_x \land (F_x \lor D_x), \]  

where \( I_x = x \) is a fully insured individual; \( A_x = x \) has attained age 62; \( F_x = x \) has filed an application for old-age insurance benefits; and \( D_x = x \) was entitled to disability insurance for the month preceding the month in which \( x \) attained the retirement age. Suppose now that counsel for the claimant, \( c \), is asked, "If you lose with respect to \( I_c \), do you lose the case? How about if you lose with respect to \( F_c \)?" A proper answer might be, "If we lose with respect to \( I_c \), we do indeed lose the case. But if we lose with respect to \( F_c \), I would still hope to persuade you that \( D_c \) is true, and if you also accept that \( I_c \) and \( A_c \) are true, then we should win."

Suppose further that the adjudicator says, "\( D_c \) seems to be a tough issue. In what circumstances must I decide it—that is, in which circumstances will it make a difference to the outcome of the case?" \( D_c \) will be decisive only if two conditions are both satisfied: (1) Given \( D_c \), the claimant is entitled to the benefit, and (2) given \( \neg D_c \), the claimant is not entitled to the benefit. The first condition is satisfied if and only if both \( I_c \) and \( A_c \) are true; the second condition is satisfied if and only if any one or more of the three propositions \( I_c \), \( A_c \), and \( F_c \) are false. Together, the two conditions are satisfied if and only if \( I_c \) and \( A_c \) are both true and \( F_c \) is false. In that circumstance only is \( D_c \) outcome-determinative.

Examined closely enough, I believe all (or at least practically all) claims have a complex structure that, like the claim under 42 U.S.C. § 402, combines conjunctive and disjunctive aspects. I cannot prove

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20 RODES & POSPESEL, supra note 3, at 236–37.
this point deductively, but I will make an inductive argument in two parts.

First, all claims, looked at closely enough, have conjunctive aspects. We might state the conditions for a claim in simple terms without any conjunctives, but doing so is almost certain to be quite uninformative. For example, we might say that

\[ T \leftrightarrow N \lor I \lor S, \] (8)

where \( T = \) Defendant is liable in tort, \( N = \) Defendant is liable for negligence, \( I = \) Defendant is liable for intentional tort, and \( S = \) Defendant is strictly liable. But obviously we want to know more. Of what does negligence consist? It still might be possible to describe this aspect of the claim without using conjunctives, but only by speaking very conclusorily: Defendant is liable for negligence if he acts negligently. Any attempt to be more informative is bound to reveal more than one requirement, at least what the defendant is supposed to have done and what impact the conduct is supposed to have had. Thus, we might say that Defendant is liable for negligence if he had a duty of care that extended to the plaintiff, he breached that duty, the breach caused an impact on the plaintiff, and the impact was injurious:

\[ N \leftrightarrow D \land B \land C \land I. \] (9)

Now, this statement of the claim for negligence is purely conjunctive. But if we care to look at it more closely, we will almost certainly be able to find disjunctive elements within it. Suppose, for example, the plaintiff is claiming that the defendant doctor committed malpractice by waiting too long to perform a Caesarean operation. Now, obviously, there are multiple injuries that might satisfy \( I; \) we might therefore replace \( I \) by \( (I_1 \lor I_2 \lor \ldots \lor I_n). \) Moreover, the plaintiff is not limited to one physiological theory as to how the negligence caused the injury. Therefore, we might also replace \( C \) by \( (C_1 \lor C_2 \lor \ldots \lor C_n). \)

Finally, consider \( D \) and \( B \) together. There are multiple ways in which an adjudicator (I am not distinguishing here between judge and jury) might conclude that these elements are true. Consider the one factor that is likely to dominate the case, time, and measure it from some marker, perhaps the first event indicating that a Caesarean might be warranted. The time when the doctor should have performed the Caesarean, and the time when he did perform it may both be in doubt. Let \( D_x = \) Defendant had a duty to perform a Caesarean before time \( x, \) and \( B_x = \) Defendant did not perform the Caesarean before time \( x. \) Then the plaintiff should prevail, so far as elements \( D \) and \( B \) are concerned, if

\[ (\exists x) (D_x \land B_x). \] (10)
This expression might be thought of as an infinitely dense disjunction, \( [(D_a \& B_a) \vee (D_b \& B_b) \vee \ldots] \), where each subscript represents an infinitesimally different point in time. Note that the later the time \( x \), the easier it will be for the plaintiff to prevail on \( D_x \) but the harder on \( B_x \). If the plaintiff can satisfy the adjudicator that there is some point in time \( x \) such that it is true both that the defendant should have performed the Caesarean before \( x \) and that he did not, then the plaintiff should prevail.

An expression like Proposition 10, indicating an infinitely dense disjunction, already suggests how the concept of elements might be broken down. We may carry the point further. With respect to some legal claims, what might appear at first as discrete elements are, I believe, better viewed as variables of a multivariate function.

I will take as an illustration the standard for granting a preliminary injunction, and as a particular example the case of *William Inglis & Sons Baking Co. v. ITT Continental Baking Co.*[^21] There, the district court denied the plaintiff's motion for preliminary relief. Putting the matter somewhat schematically, that court believed that the plaintiff must demonstrate (1) a balance of *harm* favoring the relief, and (2) *odds* of success on the merits.[^22] The appellate court reversed, holding that the district court should have considered an "alternative test" under which ["if the harm that may occur to the plaintiff is sufficiently serious, it is only necessary that there be a fair chance of success on the merits."[^23] Thus, the plaintiff should get the relief if

\[
(H_b \& O_b) \vee (H_a \& O_a),
\]  

(11)

where the subscripts \( b \) and \( a \) reflect the harm and the odds necessary to satisfy, respectively, what we might call the basic test applied by the district court and the alternative test endorsed by the appellate court. \( H_b \) reflects a lower degree of harm than does \( H_a \), but \( O_b \) (presumably equal to just over one, corresponding to the "more likely than not" standard) reflects a higher chance of success on the merits than does \( O_a \).

All well and good. But why should there be only two discrete points that justify granting the preliminary injunction? It appears that, for any given level of harm between those represented by \( H_b \) and \( H_a \), there should be a chance of success on the merits greater than

[^21]: 526 F.2d 86 (9th Cir. 1975), *reprinted in Yeazell, supra* note 7, at 349..
[^22]: *See id.* at 87. The court spoke in terms of probability, but in this case, as will soon be apparent, it becomes simpler to think at times in terms of the related concept of odds.
that represented by $O_a$ but not as great as that represented by $O_b$ that would warrant the relief. The problem can, I believe, be considered as a simple matter of decision theory, an attempt to maximize the expected utility of a decision. Under one reasonably plausible set of simplifying assumptions, preliminary relief should be granted if and only if

$$O \times (H + 1) > 2,$$  

(12)

where $1$ represents the amount of harm done by granting the preliminary relief even though the plaintiff would not ultimately prevail on the merits.\(^{24}\)

Figure 1 illustrates the point. In this diagram, the horizontal axis shows $H$, the ratio of the harm caused by denying relief if the plaintiff would ultimately prevail on the merits to the harm caused by granting relief if the defendant would ultimately prevail. The vertical axis represents $O$, the odds that the plaintiff will prevail. A rule may therefore be represented as an area within which all points indicate combinations of the two factors warranting relief. The basic test applied by the

\(^{24}\) Define $P$ to be the probability that, if the matter is fully litigated, the plaintiff will ultimately prevail on the merits. Then $1 - P$ is the probability that the plaintiff would not prevail on the merits. Also define $U(gr,\pi)$ to be the utility of granting preliminary relief if the plaintiff would ultimately prevail on the merits. Similarly, $U(den,\Delta)$ is the utility of denying relief when the plaintiff would not prevail on the merits. As a convention, we may assume that both of these utilities—representing correct decisions—are positive. $U(gr,\Delta)$ and $U(den,\pi)$ represent mistakes, and so presumably have lower utilities; as a convention, we may assume that they are each ordinarily negative.

Now, express $EU(gr)$—the expected utility of granting the relief—and $EU(den)$—the expected utility of denying relief.

$$EU(gr) = P \times U(gr,\pi) + [1 - P] \times U(gr,\Delta),$$  

(1n)

and

$$EU(den) = P \times (U(den,\pi) + [1 - P] \times U(den,\Delta)).$$  

(2n)

At the point of indecision, $EU(gr) = EU(den)$; if $P$ is any greater, $EU(gr) > EU(den)$. This means that relief should be granted if and only if

$$P \times U(gr,\pi) + [1 - P] \times U(gr,\Delta) > P \times (U(den,\pi) + [1 - P] \times U(den,\Delta),$$  

(3n)

or

$$P \times [U(gr,\pi) - U(den,\pi)] > [1 - P] \times [(U(den,\Delta) - U(gr,\Delta)].$$  

(4n)

Thus, relief should be granted if and only if

$$O \times [U(gr,\pi) - U(den,\pi)] > [U(den,\Delta) - U(gr,\Delta)].$$  

(5n)

where $O$ equals the odds that the plaintiff would ultimately prevail on the merits, and so is equal to $P/(1 - P)$.

Now, for simplicity, assume that the absolute values of $U(gr,\pi)$, $U(den,\Delta)$, and $U(gr,\Delta)$ are all 1. Further, let $H$ equal $|U(den,\pi)|$—the harm (expressed in positive terms) caused if the preliminary injunction is denied even though the plaintiff ultimately would prevail on the merits. These assumptions lead algebraically from (5n) to the rule in the text.
district court is represented by the two-sided area above and to the right of point B, with O greater than 1, indicating that the plaintiff's success on the merits is more likely than not, and H also greater than 1, indicating that more harm is done by denying the injunction even though the plaintiff would ultimately prevail on the merits than is done by granting the injunction even though the defendant would ultimately prevail on the merits. The alternative test added by the appellate court is the two-sided rule with its vertex at point A, allowing a lower probability of success on the merits if the balance of harm is more tipped to the plaintiff. The analysis here suggests that these two tests are merely continuous points along a continuum, represented by the curve C: Any point above that curve represents a combination of probability of success on the merits and a balance of harms warranting the preliminary injunction.

In other words, Proposition 12 suggests that perhaps the factors of probability (or odds) of success on the merits and balance of harms should be considered not as severable elements but rather as variables in a function, F(O,H). A little less of one variable may be compensated by a good deal more of the other. I have presented a mathematical model of such a rule, and I think doing so may sometimes be a helpful heuristic device. I do not mean to suggest that courts ought ordinarily attempt to express similar functions algebraically. Nor do I mean that legal rules ordinarily can be expressed with precision as
multivariate functions. Indeed, this exercise suggests one of the difficulties, in an "age of balancing,"\textsuperscript{25} of thinking about law as a set of rules or doctrines: If the most precise expression possible for a rule is not an expression composed of discrete elements, but instead is a function of several variables, how much of a rule can we say there really is in reality?

I am of course touching on, though I do not mean to delve very deeply into, the so-called rules-standards debate\textsuperscript{26}—which strikes me not so much as a debate as a tension, because I suppose few would say we should do altogether without bright-line rules or altogether without more flexible standards. I do mean to suggest a certain entropy in rules, or at least in many rules, that tends to break down their sharp edges. So long as rules retain their propositional nature, symbolic logic offers a useful way of thinking about their elements. But as the rules break down, and as more of one element compensates for less of another, the importance of logic recedes. In such a case, algebra may be a useful heuristic device, but it is usually no more than that.

III. A Dynamic View: Burdens and Defenses

So far, I have viewed a claim in static terms, divorced from the procedure by which it is raised and contested. Now I will discuss procedure, in the context of a claim that may be represented propositionally. I will offer here a special logic, what might be called a litigation logic.

A. Basic Premises of Litigation Logic

Classical logic is monotonic. The concept of implication is reserved for proof of proposition. Once a proposition is proved, additional premises cannot render the proof invalid. Thus, if $X \rightarrow Z$, then it must be that $[X \& Y] \rightarrow Z$ as well. In recent decades, there has been increasing interest in non-monotonic—or defeasible, default or presumptive—logics. Such logics recognize that often we draw conditions provisionally on one set of premises that we might not accept on a fuller


set of premises. Thus, even though X indicates Z, X & Y may not do so. Non-monotonic reasoning has become a matter of great interest not only to formal logicians, especially those engaged in normative, or deontic, logic, but also to epistemologists, including those who

27 See, e.g., Drew McDermott, Nonmonotonic Logic II: Nonmonotonic Modal Theories, 29 J. Ass'n Computing Machinery 35, 53 (1982) (saying that the term nonmonotonic "refers to first-order theories in which new axioms can wipe out old theorems"); Terry Winograd, Extended Inference Modes in Reasoning by Computer Systems, 13 Artificial Intelligence 5, 5–6 (1980) ("In mathematics, one does not accept a conclusion unless it can be 'proved' according to the laws of inference and the initially accepted axioms. In real life, we are faced with limitations of our knowledge and demands for action. . . . [Non-monotonic reasoning] must draw a conclusion which, on the basis of further information, it would later reject. Ordinary notions of deduction do not allow non-monotonicity. If A is provable from a set of initial axioms, then no additional axioms can ever cause [-A] to be provable, unless the theory is inconsistent.").

28 See, e.g., Nicholas Rescher, Plausible Reasoning (1976) (developing a formal system for dealing with cognitive dissonance created by inconsistent information); Judea Pearl, Probabilistic Semantics for Nonmonotonic Reasoning, in Philosophy and AI: Essays at the Interface 157 (Robert L. Cummins & John L. Pollock eds., 1991). Some of the formal explorations of non-monotonic logic have been presented as part of a logic of theory change. See, e.g., David Makinson, How to Give It Up: A Survey of Formal Aspects of the Logic of Theory Change, 62 Synthese 547 (1985); Carlos E. Alchourrón et al., On the Logic of Theory Change: Partial Meet Contraction and Revision Functions, 50 J. Symbolic Logic 510 (1985); see also Peter Gardenfors, The Dynamics of Belief: Contraction and Revisions of Probability Functions, 5 Topoi 29 (1986) (generalizing upon prior work with Alchourron and Makinson, modeling states of belief by probability functions). I am inclined to agree with those who argue that intuitions about how defaults work are at least often captured better by speaking of flexible inference rules within a theory rather than of theory change. See McDermott, supra note 27, at 55.

29 See, e.g., Defeasible Deontic Logic (Donald Nute ed., 1997); Marvin Belzer, Reasoning With Defeasible Principles, 66 Synthese 135 (1986). Deontic logic is often referred to as logic of obligation or logic of norms, involving normative expressions such as obligation, duty, permission, and right. See Dagfinn Follesdal & Risto Hilpinen, Deontic Logic: An Introduction, in Deontic Logic: Introductory and Systematic Readings 1, 1 (Risto Hilpinen ed., 1971). The logic I am presenting here is not limited to deontic purposes; it is meant to express the interrelation of propositions material to legal conclusions, and can be used for both descriptive and prescriptive purposes.

30 See, e.g., John L. Pollock, Defeasible Reasoning, 11 Cognitive Science 481 (1987); John L. Pollock, OSCAR: A General Theory of Rationality, in Philosophy and AI, supra note 28, at 189; see also Introduction to id. at 5 ("Of the most significant advances in epistemology in the second half of the twentieth century has been the recognition that most reasoning is defeasible. . . . Researchers in artificial intelligence made the same discovery independently . . . .").
study pragmatic or everyday argument,\textsuperscript{31} cognitive psychologists,\textsuperscript{32} and scholars of artificial intelligence.\textsuperscript{33}

\textsuperscript{31} See, e.g., Douglas N. Walton, \textit{Plausible Argument in Everyday Conversation} (1992). Walton, a participant in this symposium, notes that "a new awakening of interest in evaluating arguments in relation to how they are really used in conversational context" has resulted in "the advent of the informal logic movement," which is concerned with "the practical study of how arguments are used in everyday reasoning." \textit{Id.} at 2. Such use bears a close relation to the use of argument in law. As Walton says, in everyday conversations,

typically, disputes cannot, given practical constraints, be definitively resolved by clear, obtainable evidence that proves, beyond doubt, that a proposition is true or false. Instead, such arguments are typically based on presumptions, and the argument is good or bad, successful or unsuccessful, insofar as a \textit{prima facie} case can be made for or against it.

\textit{Id.} Much the same could, of course, be said about legal argument. Walton's "fundamental thesis" is that argument in everyday conversations needs to be evaluated by standards that are lower than the inductive or deductive standards typically used in formal logic. \textit{Id.} at 3. My aim in this part of the article is to make essentially the same type of assertion with respect to legal argument. Interestingly also, Walton says that presumptive reasoning of the type he presents is best considered "in a context of a dialogue." \textit{Id.} at 4. In this respect, too, it bears some similarity to the type of logic I am presenting here, which is meant to represent an argument conducted by two litigants. But then our purposes diverge. Walton is concerned with participants engaged in "reasonable argument," meaning that they adhere to standards under which the conversation might shed light on the matter. \textit{Id.} at 6. My purpose is to represent an argument between adversaries. Under the logic presented here, evaluation of the argument—determination of what action is to be taken—is critical and is performed by a third entity, the adjudicative system. In Walton's system, by contrast, evaluation plays a lesser role and has more modest goals. \textit{See id.} at 288 ("The proper question of evaluation is ... whether or not the argument contributes to the critical discussion.").

\textit{See also} Allan Collins & Ryszard Michalski, \textit{The Logic of Plausible Reasoning: A Core Theory}, 13 Cognitive Science 1 (1989). Rescher, \textit{ supra} note 28, may also be classified as a work of pragmatic reasoning; much of his concern is in application of his theory to practical issues of inconsistent information.


\textsuperscript{33} See, e.g., McDermott, \textit{ supra} note 27; Winograd, \textit{ supra} note 27; R. Reiter, \textit{A Logic for Default Reasoning}, 13 Artificial Intelligence 81 (1980). The Winograd and Reiter articles are part of an important \textit{Special Issue on Non-Monotonic Logic}, 13 Artificial Intelligence no. 1, 2 (1980). For another significant special issue, see \textit{Default Reasoning}, 1 Minds-and-Machines no. 4 (1991). \textit{See also} Donald Nute, \textit{Preface to Defeasible Deontic Logic}, \textit{ supra} note 29, at vii (noting that "despite the long-time interest in defeasible reasoning within the philosophical community, ... the investigation of defeasible or 'nonmonotonic' reasoning received a huge impetus when the artificial intelligence community discovered it"); Timothy R. Colburn, \textit{Heuristics, Justification, and Defeasible Reasoning}, 5 Minds & Machines 467, 484-86 (1995) (discussing the role of defeasible rules in justifying "qualitative heuristics" and contending that ["s]ince the modeling of defeasibility is critical for the creation of intelligent programs, artifi-
Interestingly, though jurists are prominent among those who might be expected to be "natural clients" of the deontic logician, so far as I am aware no prior efforts have been made to present a relatively simple logic that is closely related to classical propositional logic and yet captures the defeasibility of legal conclusions. In making this preliminary attempt to do so, I believe I am, mercifully, relieved from having to rely heavily on the rich, complex, and varied literature on non-monotonic logic. This is in part because "all the work on nonmonotonic reasoning in the past twenty years or so has not produced a convergence of systems or even of intuition," and in part because my aim is a logic that is both accessible and designed for the distinctive problems of litigation and legal doctrine. I will use a special symbol, \( \Lambda \), which might be thought of as an arrowhead, a cropped form of the arrow of implication. This symbol may be read as "presumptively implies." Thus, \( X \Lambda Y \) means, "If we know \( X \), then \( Y \) is presumptively true, so that if at the end of the case the only material information we have is \( X \), then we will act as if \( Y \) is true." Thus, \( X \) is

cial intelligence researchers attempting to do knowledge representation must do some epistemology in order to understand what they are modeling”).

34 See Donald Nute & Xiaochang Yu, Introduction to Defeasible Deontic Logic, supra note 29, at 1. It appears that the earliest known use of the term “defeasible” in the philosophical literature is by H.L.A. Hart. See Nute, supra note 33, at vii.

35 There have been some important prior works in the legal literature that, in analyzing litigation or doctrinal problems according to logical methods, have incorporated non-monotonic aspects into the analysis. Much of the work of my colleague Layman Allen fits this description. See also, e.g., Saunders, supra note 6. A particularly well known work that might be deemed to fit here is John Henry Wigmore, The Science of Judicial Proof (3d ed. 1937). Several notable recent works have self-consciously followed in Wigmore’s footsteps. See Terence Anderson & William Twinning, Analysis of Evidence: How to Do Things With Facts (1991); Joseph B. Kadane & David A. Schum, A Probabilistic Analysis of the Sacco and Vanzetti Evidence (1996); Peter Tillers & David A. Schum, Charting New Territory in Judicial Proof: Beyond Wigmore, 9 Cardozo L. Rev. 907 (1988). But the aim of all these works is much different from that of the discussion here, as described in the text above. Wigmore and those following him, for example, are primarily interested in organizing proof of a case according to systematic methods.

36 Nute, supra note 33, at vii.

37 Throughout this Part, I use the language of burdens and presumptions. I do not mean the use of these terms to be limited here to the narrow sense in which they are most rigorously used in the law of evidence. See, e.g., 2 McCormick on Evidence §§ 356, 342 (John William Strong ed., 4th ed. 1992).

38 Howard Pospesel has made the reasonable-sounding suggestion that I attempt to state a nonverbal definition of the \( \Lambda \) operator. Perhaps a logician could state such a definition; I have not been able to state one that satisfies me. But below, in notes 46 and 47, infra, I offer what is perhaps the next best thing, a catalogue of various basic
not airtight proof of $Y$; it gives a basis to act as if $Y$ is true unless we learn more. We may take as a rule that
\[ ((X \supset Z) \land [(X \land Y \supset -Z) \land X \land Y] \supset -Z. \] (13)
That is, if one presumptive implication has a set of premises that is a subset of the set of premises of another presumptive implication, and the fuller set of premises is demonstrated, then the latter presumptive implication—the one operating on more information—is the one that governs.\(^{39}\)

Expression 13 is, in substance, a standard aspect of non-monotonic logics. But now I will add a feature that is more distinctively addressed to litigation. Let $L$ represent the proposition that the defendant is liable. Because the judicial system must act at the end of the litigation either as if $L$ is true or as if $L$ is false, we can adopt the rule that
\[ (X \supset L) \lor (X \supset -L). \] (14)
That is, a given set of premises presumptively implies one outcome or the other, $L$ or $-L$; at the end of the case, whatever the factual findings may be, the system must decide one way or the other.

The notion of presumptive implication is, I think, useful in two ways, suggesting on the one hand how the law may operate without complete articulation and on the other how complete articulation might be impossible to accomplish. First, presumptive implication expresses the idea that a given set of premises may yield a particular result without any necessity for a logical proof that those premises imply that this result must prevail whatever else must be learned. Second, presumptive implication is compatible with the idea that, at least in some areas of the law, full implication of this more stringent type is an impossibility. This is a concept that is familiar, and troubling, to many law students: Try to come up with a rule of law that is complete, subject to no qualifications or exceptions. It is hard to do. There will almost always be a "spitting cobra" type of exception.\(^{40}\)

\(^{39}\) If $(X \land Y) \supset Z$ and $(X \land W) \supset -Z$, then $W \land X \land Y$ could presumptively imply either $Z$ or $-Z$; the situation is not determinate on the information given.

\(^{40}\) I am referring to the following argument by my friend and colleague Brian Simpson:

Suppose we have a rule that the children are always to have a bath before going to bed, which seems a sensible enough rule. Obviously there need to be exceptions. For example, it would be silly to insist on a bath if one of the children was critically ill with pneumonia. We could try to think of all the exceptions in advance, but the task is hopeless, as becomes clear on the day
I do not mean to suggest that implication of the classical type is foreign to the law. For example, if one proposition is equivalent to another according to principles of classical implication, then either may be substituted for the other in a presumptive implication. Perhaps more obviously, a crime may be defined by statute so that the defendant cannot be deemed to have committed the crime unless the statutory elements are satisfied. Nevertheless, there is a wide range of areas in which the law does not operate in a crisp, definitional matter. Indeed, the fact that a given body of law is definitional does not necessarily eliminate a role for presumptive implication, for the elements of the definition may be quite vague and conclusory, and presumptive implication may operate at the level of these elements even if it does not at the higher level of the claim as a whole.

B. Litigation Logic in Operation

Now, let $O$ equal all the information about the world that the jury is entitled to bring into the courtroom. Then we may say that

$$O \not\rightarrow L.$$  \hspace{1cm} (15)

That is, based on nothing but the information that the jury is entitled to carry from the outside world, the defendant is not liable. Notice that this rule—the presumption of innocence, or the initial allocation of the burden of production on the plaintiff—does not amount to a rule that the defendant is not liable. It simply says that if no facts are established in litigation the defendant wins.

Now the plaintiff brings a complaint. I will use an example used by Rodes and Pospesel, though with somewhat different notation. The plaintiff alleges

$$B & F,$$  \hspace{1cm} (16)

where $B$ = defendant bought a horse from plaintiff, and $F$ = defendant failed to pay for the horse.

The complaint itself does not have to contain an argument of law. Implicit in the complaint, however, is the legal contention that

$$(O & B & F) \not\rightarrow L.$$  \hspace{1cm} (17)

when, through an accident to a travelling circus, we find a spitting cobra in the bath. Nobody would have ever thought of that possibility, and provided for it by the spitting cobra exception.


41 See infra note 47.

Now, assuming proper jurisdiction and venue and all that, the defendant has three basic choices in responding to this complaint. First, he may move to dismiss it for failure to state a claim on which relief may be granted. This amounts to a denial of Proposition 17, and so according to the rule of Proposition 14, an assertion that

\[(O \& B \& F) \uparrow -L.\]  (18)

Second, the defendant may deny an essential factual allegation of the complaint. That is, he may deny the truth of Proposition 16, asserting

\[-(B \& F).\]  (19)

Such a minimalist denial will in itself probably be considered too uninformative. The defendant must identify which of the assertions made by the plaintiff he intends to deny. Conventional logic shows that the defendant will effectively assert Proposition 19 if he asserts either of the propositions \(-B\) or \(-F\); he may also assert both.

Third, he may assert an affirmative defense. This is a new issue that he raises and that, he contends, defeats liability even if the assertions of the complaint are true. Thus, he may assert \(M\), the proposition that he was a minor when he bought the horse. In doing so, he will be prepared to make—if challenged—the legal argument that

\[(O \& B \& F \& M) \uparrow -L.\]  (20)

Note that these three responses are mutually compatible but not mutually dependent. That is, the defendant could (as a logical matter and under modern practice) make any one of the three whether he makes either or both of the other responses. For example, the defendant could contend, “It is not true that I bought the horse from you and failed to pay for it. Moreover, even if you were able to prove those allegations they would not be sufficient for relief.” The relation

43 I do not believe the defendant needs to be either more particular or more general than the plaintiff. Thus, I do not accept Rodes and Pospesel’s assertion that if the plaintiff asserts that defendant shot plaintiff with a .38 caliber revolver, the “correct response” is “Defendant denies that he shot plaintiff with a .38 caliber revolver or any other weapon.” RODES & POSPESEL, supra note 3, at 216. Even if in a prior era pleading law generally required such a response, a matter on which I am unsure, I do not believe that it does so in the modern era. The defendant should not be forced to do the plaintiff’s conceptual thinking and to articulate his case, to say in effect, “Well, you didn’t get it quite right, but I understand what category you were thinking about, and you may have a case in the general vicinity of what you alleged.” If the plaintiff alleged that defendant shot him “with a .38 or with some other weapon,” then the defendant would be required to answer more comprehensively; part of such a response might be an assertion that the allegation of “some other weapon” was too vague.
between the motion to dismiss and the affirmative defense is particularly interesting. The affirmative defense may be raised without making the motion to dismiss. In this case, such a stance would effectively concede the implicit assertion of Proposition 17. This is close to the old-fashioned confession and avoidance: "I concede that purchase of the horse and failure to pay would, without more, be sufficient for relief. But even so, and even if the plaintiff proves everything he's alleged, the additional factor that I've raised, minority, is enough to defeat his claim."

Alternatively, the affirmative defense might be a back-up to the motion to dismiss. That is, the defendant need not confess to avoid. He might say, "I contend that purchase of the horse and failure to pay would not, without more, be sufficient for relief. But even if I'm wrong about that, and even if the plaintiff proves everything he's alleged, the additional factor that I've raised, minority, is enough to defeat his claim." In making this argument, the defendant might suggest that at least one reason the complaint is inadequate is that it did not plead \(-M\), the negation of the very proposition raised by the affirmative defense.\(^4\) The argument would then have this nature:

An essential element of plaintiff's claim, on the theory that he has presented, is that the purchaser has achieved the age of consent.\(^5\)

\(^4\) Stating the argument like this might make it appear that the defendant is being unreasonable by asking that the plaintiff be required to plead and prove a negative. But as Kevin Saunders has argued in a perceptive essay, it is not proving a negative that is difficult. See Kevin W. Saunders, *The Mythic Difficulty in Proving a Negative*, 15 SETON HALL L. REV. 276 (1985). Any proposition framed in negative terms might, with a change of wording, be turned into a positive. Thus, for example, "Defendant was not a minor at that time" might be rephrased as "Defendant had achieved the age of consent by that time." Similarly, "Plaintiff was contributorily negligent" may be rendered "Plaintiff failed to exercise due care." Saunders also points to another factor that has much more significance in assigning the burdens of proof: It is generally easier to prove (and more sensible to require pleading of) an existential proposition than a universal one. *Id.* at 281. Thus, it is probably easier to prove "Plaintiff herself failed to exercise due care in that just before the collision she was eating a sandwich while driving" than to prove "Plaintiff exercised due care at all material times and in all material respects." Note that the negation of an existential proposition is a universal, and vice versa.

\(^5\) This sentence may require some explanation. I have spoken about various premises presumptively implying liability or its negation; I have not spoken of liability as presumptively implying any proposition or set of propositions. As I have suggested above in discussing the difficulty of complete articulation, the theory on which the plaintiff pleads is presumably not the exclusive basis on which liability might be found; for example, a far different set of propositions might warrant judgment for the value of the horse on a tort theory. Given that there may be other theories of liability possible, the defendant probably cannot argue persuasively that \(-M\) is logically essen-
The plaintiff hasn’t pleaded that issue, so the complaint should be dismissed. But in case I’m wrong about that—so that this issue is an affirmative defense rather than one on which the plaintiff has the burden of pleading—I have pleaded minority. Even if the plaintiff proves everything he’s alleged, this factor is enough to defeat his claim.

In other words, even while asserting Propositions 17 and 19 as well as M, the defendant might be willing to concede, at least for the purpose of argument, that

\[(O \& B \& F \& -M) \ni L. \quad (21)\]

Notice that the motion to dismiss and the denial join issue with the plaintiff, denying the truth of some proposition that, explicitly or implicitly, the plaintiff has asserted. The affirmative defense does not. Thus, the plaintiff may respond to the affirmative defense in the same ways that the defendant responded to the complaint. He may contest its legal validity, denying the truth of Proposition 20. He may deny its factual premise M. And he may affirmatively introduce another factor that, he contends, presumptively establishes liability even if the facts underlying the affirmative defense are made out. Thus, the plaintiff may assert N, that the horse was a “necessary,” and contend that, whether or not Proposition 20 is true,

\[(O \& B \& F \& M \& N) \ni L. \quad (22)\]

And so it might go on. Eventually, the judicial system will adjudicate the truth of factual premises and the legal validity of presumptive implications. According to the rule of Proposition 13, the valid presumptive implication with the fullest set of premises demonstrated to the satisfaction of the system would then prevail.

The logic I have set out here does not in itself prescribe which presumptive implications—which potential rules of law—are valid.\(^\text{46}\) In some cases, often in conjunction with principles of classical logic, it

\(^\text{46}\) One of the factors limiting the use of presumptive logic to draw inferences about the validity of legal arguments, rather than merely to state the arguments clearly, is that many of the principles governing implication in classical logic do not hold in presumptive logic, or have weak counterparts.

Consider the principle of *modus ponens*. In classical logic, this principle prescribes that from \((X \rightarrow Y) \& X\), Y follows inevitably. Clearly, from \((X \ni Y) \& X\), Y...
can be of assistance in this respect, indicating that if one particular presumptive implication is valid then another particular one is or is not.\textsuperscript{47} But the determination of which presumptive implications are valid is primarily a matter for the lawmakers aspects of the adjudicative system, and they do not operate predominantly on the basis of logic.\textsuperscript{48} The main value of the logic presented here is not that it leads does not follow inevitably, but we can make the weaker statement that \([ (X \triangleright Y) \& X] \triangleright Y\).

Nor does the principle of contraposition hold; from \(X \triangleright Y\), it does not necessarily follow that the contrapositive, \(-Y \triangleright -X\), is true. Thus, if \(X = \) the driver was driving under 40 miles per hour and \(Y = \) the driver was driving legally, it may be that \(X \triangleright Y\) is true but \(-Y \triangleright -X\) is not.

Similarly, transitivity does not hold; from \(X \triangleright Y\) and \(Y \triangleright Z\), we cannot infer that \(X \triangleright Z\) (though this relation would often be true). For example, suppose that \(X = T\) was alive on January 1, 1980, \(Y = T\) was alive on January 1, 1986, and \(Z = T\) was alive on January 1, 1992, and that a governing principle is that it is not generally presumed that a given person was alive on any given date, but that proof that the person was alive on a date no more than seven years before the given date does create such a presumption. Then \(X \triangleright Y\) and \(Y \triangleright Z\), but it is not true that \(X \triangleright Z\).

\textsuperscript{47} Here are some principles, which I believe may be taken as premises of the system of defeasible logic presented here. Some of these show how the implication operator \(\rightarrow\), of classical logic may interact with the \(\triangleright\) operator used here.

If \(X \rightarrow Y\), then it must be that \(X \leftrightarrow Y\), because classical implication is a stronger condition than presumptive implication. By contrast, from \(X \triangleright Y\), it does not follow that \(X \rightarrow Y\).

If \(X \triangleright Y\) and \(Y \rightarrow Z\), \(X \leftrightarrow Z\). Similarly, if \(X \triangleright Y\) and \(X \leftarrow Z\), \(Z \triangleright Y\). But if \(X \triangleright Y\) and \(Z \rightarrow X\), it does not necessarily follow (though it usually would) that \(Z \triangleright Y\). Thus, if \(X = \) a prospective witness is under 4 years of age, \(Y = \) the prospective witness should not be allowed to testify, and \(Z = \) the prospective witness is a very advanced child 3 1/2 years old, then even if \(X \triangleright Y\), it may well be that \(Z \triangleright -Y\).

From \((X \& Y) \triangleright Z\), I believe it follows necessarily that \(X \triangleright (Y \triangleright Z)\). But the converse does not hold: From \(X \triangleright (Y \triangleright Z)\), it does not necessarily follow that \((X \& Y) \triangleright Z\). Here is a counterexample showing why not. Suppose there are several large bags, one of which is marked \(A\). In bag \(A\) are a number of urns, ten of which are marked \(B\). In each urn marked \(B\) there are ten balls; in seven of those urns, six of the balls are marked \(C\) and in the remaining three urns none of the balls are marked \(C\). A player picks a bag, then an urn from the bag, then a ball from the urn. Now let \(A = \) bag \(A\) is picked, \(B = \) a \(B\) urn is picked, and \(C = \) a \(C\) ball is picked, and for purposes of this problem assume that a conclusion is deemed to be implied presumptively from a predicate if it is more likely than not given the predicate. Then \(A \triangleright (B \triangleright C)\) is true—if bag \(A\) is picked, it is probable that if a \(B\) urn is picked it will be probable that a \(C\) ball will be picked. But \((A \& B) \triangleright C\) is not true—if bag \(A\) is picked and then a \(B\) urn, it is more likely than not that a \(C\) ball will \textit{not} be picked. The intuitive difference is that, while the second expression has only one statement of uncertainty, the first statement has two, making it weaker and therefore more easily satisfied.

\textsuperscript{48} Often, the problem will have the structure suggested in note 39, \textit{supra}, in which one party presents the presumptive implication \((X \& Y) \triangleright Z\) and the other party presents the presumptive implication \((X \& W) \triangleright -Z\). If both presumptive implications
to conclusions but that it helps set out clearly the nature of a legal contention, dispute or doctrine.

C. The Logics Compared

I have now sketched out in some detail, though not with formal rigor, a form of presumptive logic. For several related reasons, I believe it better represents the logic of litigation than does the classical logic presented by Rodes and Pospesel.

The nature of legal arguments is usually presumptive. No matter how stridently a lawgiver may proclaim, "If you are a child and it is evening, you must take a bath," there is an unspoken qualification, "So long as you don't have pneumonia, and there isn't a spitting cobra in the bathtub, and there isn't any other reason that I haven't yet articulated as to why you needn't have a bath." The presumptive assertion of law is all that one making a legal argument needs to claim, and all that he can claim persuasively. (At least that is presumptively true.)

Because it is based on such presumptive assertions, the presumptive logic presented here captures a critical aspect of the dynamics of litigation, the shifting of burdens. The plaintiff begins with a presumption against him, expressed by Proposition 15. If he fails to persuade the adjudicator that Proposition 16 is true, or that Proposition 17 is legally correct, this does not mean that he loses the litigation, but only that he has failed to overcome that presumption. If he does persuade the adjudicator with respect to both Proposition 16 and Proposition 17, then he is presumptively entitled to relief. This does not amount to a logical proof of L, but it shifts the burden to the defendant to come up with a reason why, even though the plaintiff's legal and factual assertions are correct, the plaintiff is not entitled to relief.

The classical logical approach, by contrast, seeks logical proofs of a proposition. This approach is overly brittle in the litigation context: If a proposition is not proven, then a judicial resolution cannot be reached by virtue of it. Consider first the significance of this difficulty at the outset of the litigation. Without any premises, neither L nor -L can be proven. Thus, there is nothing in the classical approach comparable to Proposition 15, which expresses the fundamental allocation of the initial burden of pleading.

are valid, and if in addition W, X, and Y are all taken to be true—so that the overlapping predicates for both presumptive implications are established—the situation is not logically determinate on this information; the determiner of law must decide which presumptive implication, Z or -Z, (W & X & Y) follows from Z or -Z.

49 See Simpson, supra note 28, at 80.
Now suppose that the plaintiff pleads a set of premises that he contends is sufficient for relief. First consider a denial by the defense. If the denial is accepted as accurate—so that the factual premises asserted by the plaintiff are not completely true—this does not prove that relief is unwarranted unless it is assumed that the theory raised by the plaintiff is the exclusive basis for relief. And so Rodes and Pospesel are, in my view, forced to make overly extensive use of the maxim *inclusio unius est exclusio alterius*—the inclusion of one thing is the exclusion of another. They recognize that this principle is only a principle of legal interpretation, not a logical principle. But they make the dubious contention that it expresses the usual tendency of the law, and they rely on it heavily to demonstrate that under given circumstances the defendant cannot be liable. A more parsimonious approach avoids the question of whether a given doctrine is the exclusive source of relief—unless the exclusivity argument is raised to show that another purported source of relief is spurious. Instead, this approach relies on the fact that the plaintiff has not raised any other theory justifying relief, so that if the theory he has raised is not factually supported he has not overcome the initial presumption of non-liability.

A rather similar point holds with respect to defendant’s motion to dismiss for failure to state a claim. It may distort matters to treat the argument supporting the motion, if accepted, as a demonstration of non-liability. Consider Rodes and Pospesel’s rendition, under classical logic, of a declaration of law in the old case of *Asseltyne v. Fay Hotel*:

\[(x) [Ix \to (y) (Byx \to -Lxy)], \tag{23}\]

where \(Ix = x\) is an inn, \(Byx = y\) is a boarder in \(x\), and \(Lxy = x\) is liable for the property of \(y\) destroyed in a fire at the inn. In English, Proposition 23 reads, “An inn is not liable for the property of its boarders

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50 RODES & POSPESEL, *supra* note 3, at 239. I think this statement is dubious because so often the maxim is violated. *Cf.* Karl N. Llewellyn, *Remarks on the Theory of Appellate Decision and the Rules or Canons About How Statutes Are to Be Construed*, 3 Vand. L. Rev. 395, 405 (1950) (noting that alongside the *exclusio unius* maxim is another pointing in the opposite direction: “The language may fairly comprehend many different cases where some only are expressly mentioned by way of example.”). For example, Rule 11 provides for sanctions for irresponsible conduct that gratuitously creates extra litigation. But so does 28 U.S.C. § 1927 (1994). And if the particular case does not satisfy the requirements of either of these provisions, the court may still impose a sanction as a matter of its inherent power. *See* Chambers v. NASCO, Inc., 501 U.S. 32 (1991).

51 23 N.W.2d 357 (Minn. 1946).

52 RODES & POSPESEL, *supra* note 3, at 200.
destroyed in a fire at the inn.” But plainly this is too absolute a state-
ment. An inn may be liable for the property of its boarder if, for ex-
ample, the inn commits an intentional tort that results in the
 destruction of the boarder’s property; if the fire in Asseltyne were com-
m ited for the purpose of destroying Miss Asseltyne’s property, the
hotel presumably would have been liable for the loss (at least absent
some extenuating circumstance—such as that the destruction was nec-
essary to prevent some grave pestilence). A more accurate statement
would be, “The fact that a person is a boarder of a hotel is not in itself
sufficient to create liability for the hotel.” In the terms I have used,

I & B ⊧ -L. (24)

This leaves room for the possibility that other facts will presumptively
establish L.

Finally, consider affirmative defenses. Rodes and Pospesel at-
tempt to represent these by using the classical technique of distinguo,
in which one party to a disputation concedes an assertion made by the
other party under one assumption but denies it under a contrary as-
sumption. Thus, the defendant in the horse-sale case, in response to
the plaintiff’s assertion that (O & B & F) → L, might concede that

(O & B & F & -M) → L, (25)

but deny that

(O & B & F & M) → L, (26)

assert affirmatively that

(O & B & F & M) → -L, (27)

and also assert that M is in fact true.53

This is an interesting approach, but I do not believe it fully cap-
tures the nature of an affirmative defense.54 First, to make the affir-

53 Proposition 27 does not follow logically from the denial of Proposition 26, but
Rodes and Pospesel present both (in different notation) as part of the defendant’s
argument. See RODES & POSPESEL, supra note 3, at 230–31 (lines 4 and 7B). Similarly,
they include as part of the plaintiff’s argument both (a) a negation of the proposition
that the defendant is not liable if the defendant bought the horse while a minor and
the horse is a “necessary” and (b) an affirmative assertion that under these circum-
stances the defendant is liable. Id. at 231 (lines 8 & 11B).

54 Rodes and Pospesel contend that a statement should be distinguished rather
than denied if any of three circumstances holds. At least two of them are significant
for affirmative defenses: “Although true, . . . [the statement] will be misleading unless
additional circumstances are brought forward,” and “[The statement] is subject to an
exception which law or common sense requires to be specifically brought forward
. . . .” Id. at 232. I do not find this articulation helpful. Much more straightforward, it
seems to me, is to say that a defendant should assert a given proposition as an affirm-
itive defense if, at least arguably, the plaintiff would be presumptively entitled to judg-
ative defense, the defendant need not concede the point reflected by Proposition 25. The defendant might, as I have contended above, suggest that if the plaintiff were able to prove \(-M\), along with \(B\) and \(F\), he would (presumptively) prevail, but there is no need that he do so. The defendant might contend that even the premise of Proposition 25 is an insufficient basis for liability; or he might contend that liability cannot be *proven* because another affirmative defense might defeat it.

Second, to assert the affirmative defense, the defendant need not make so strong an assertion as Proposition 27. The defendant need only make a presumptive assertion. This acknowledges that, even if \((O \& B \& F \& M)\) is true, liability may still be made out; the plaintiff may raise an issue to defeat the effect of \(M\), or he may be able to make out a different theory of liability altogether.

Finally, although together Propositions 25 and 27 indicate that, assuming the truth of \(B\) and \(F\), the issue of \(M\) is critical to the action, these propositions give no sense of the burden of pleading. In other words, these propositions act as a sort of melange of a motion to dismiss and an affirmative action. They say in effect, "If \(B\), \(F\), and \(-M\) are true, then he wins, but if \(M\) is true I win even if \(B\) and \(F\) are true." That is fair enough—but then is the argument that the complaint is insufficient because it fails to plead \(-M\), or that, although the complaint is sufficient, if the defendant pleads and proves \(M\) then liability is defeated? I have shown in Section B how the presumptive approach quite neatly separates these arguments.

Perhaps able users of classical logic, like Rodes and Pospesel, could address these problems—in part, perhaps, by including carefully crafted limitations in the universe to which the application applies. I suspect such attempts would significantly complicate the application of the logic. The problem, I believe, is that classical logic, with very broad applicability, is aimed at proofs of propositions. Thus, it is not well designed for litigation, at least litigation in the common law style, in which the question is not, "Is the defendant's liability *logically proven*?" but "Given what we know and what has happened, should we impose liability on the defendant?" The presumptive logic I have presented here, a much more specialized tool than classical logic, is designed expressly to reflect the dynamics of our litigation system.

*ment under a given set of premises that the plaintiff has pleaded, but would not be presumptively entitled to judgment if that set of premises is supplemented by the proposition in question.*
IV. Conclusion

I do not believe that logic often prescribes results in legal cases. By the time we are able to make a statement of the form, “If a set of premises A is true, then a set of consequences B must follow,” the hard work of making law has already been done. But logic can set constraints on legal doctrine or discourse; a doctrine or argument that can be shown to be internally inconsistent as a logical matter will lack persuasive power. Because we often divide a claim into separate elements, symbolic logic can be a useful heuristic in analyzing the structure of the claim; in other cases, in which the claim more resembles a function of multiple variables, this usefulness is lessened. In some settings, the heuristic benefit of logical analysis can be enhanced by using a logic of the sort presented here, reflecting the defeasibility of much legal doctrine.