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RIGHT₁, RIGHT₂, RIGHT₃, RIGHT₄,

AND HOW ABOUT RIGHT^T?

LAYMAN E. ALLEN

INTRODUCTION

Careful communication is frequently of central importance in law. The language used to communicate even with oneself in private thought profoundly influences the quality of that effort; but when one attempts to transmit an idea to another, language assumes even greater significance because of the possibilities for enormously distorting the idea. Word-skill is to be prized. Few have expressed this more aptly or succinctly than Wesley N. Hohfeld:¹

... [I]n any closely reasoned problem, whether legal or non-legal, chameleon-hued words are a peril both to clear thought and to lucid expression.

Such a proposition should not be controversial among lawyers and legal scholars. In any event, the starting assumption of this article is that careful communication is sometimes useful. To facilitate such communication where it is deemed desirable, the discussion that follows is aimed at helping to clarify one of the fundamental concepts in legal discourse — namely, the concept of *right*. This task will be approached in the spirit in which Felix S. Cohen explored such ideas² — not asking,

What does the term ‘right’ really mean?

but rather,

How is it most useful to define it?

Certainly, it is still worth while to do what we can to make less haunting the chilling, more-than-half-century-old reminder of John Chipman Gray:³

... The student of Jurisprudence is at times troubled by the thought that he is dealing not with things, but with words, that he is busy with the shape and size of counters in a game of logomachy, but when he fully realizes how these words have

been passed and are still being passed as money, not only by fools and *on fools*, but by and on some of the acutest minds, he feels that there is work worthy of being done, if only it can be done worthily.

DISTINGUISHING VARIOUS SENSES OF 'RIGHT'

In his classic effort to help clarify legal discourse by specifying for it a set of "lowest common denominators," Hohfeld indicates prior judicial recognition that the term 'right' is used indiscriminately and ambiguously to denote a wide variety of legal relations.⁴ Sometimes 'right' is used to indicate a *privilege* to do something. On other occasions its reference is to a *power* to create some legal relationship. Still other times it is used to show that someone has *immunity* from having his legal status changed in some way. Mostly, however, it is used to refer to *right* in the strict sense of somebody else's obligation to do something for the right-holder.⁵ For each of these four different senses of 'right', Hohfeld stipulated a different term:

Right₁ right (in strict sense)

Right₂ privilege

Right₃ power

Right₄ immunity

Right₁. The definite and appropriate meaning that Hohfeld stipulated 'right' (in the strict sense of a legally enforceable claim) should refer to is the correlative of 'duty'. He gave as example:⁶

... [I]f X has a right against Y that he shall stay off the former's land, the correlative (and equivalent) is that Y is under a duty toward X to stay off the place.

Hohfeld, in effect, thus specified 'right' to refer to a three-term relationship between two persons and an action—the right-holder, the other party, and an act of the other party. To say that x has a (legal) right that p shall be done by y is the same as to say that it is obligatory that p be done by y for x, and that the legal system will enforce the obligation.

Consider a statement like 'The ball is red'. This statement attributes the property red (R) to the ball (b). An abbreviated way of writing it is 'Rb'. Just as 'Rb' is an abbreviated way of writing 'The ball is red', so too is '(Rpy)x', or more briefly, 'Rpyx' an abbreviated way of writing 'x has a right that p be done by y'.

Right₂. Hohfeld emphasized the importance of distinguishing the concept of privilege from the concept of right, and he reserved the

term 'privilege' to refer to what one person was permitted to do as far as some other person was concerned. Continuing the prior example, he states:⁷

. . . [W]hereas X has a *right* or *claim* that Y, the other man, should stay off the land, he himself has the *privilege* of entering on the land; or, in equivalent words, X does not have a duty to stay off. The privilege of entering is the negation of a duty to stay off.

So, 'privilege' also is also, in effect, specified by Hohfeld to refer to a three-term relationship between two persons and an action — the privilege-holder, the other party, and an act of the privilege-holder. To say that x has a (legal) privilege to do p as far as y is concerned is the same as to say that it is permissible for p to be done as far as y is concerned by x and that the legal system will not enforce any attempt through litigation by y to prevent x from doing p.

Right₃. For Hohfeld 'power' is most usefully reserved to refer to the change in legal relations that results from some "superadded fact or group of facts which are under the volitional control of one or more human beings." The person (or persons) whose volitional control is paramount has the (legal) power to effect the particular change of legal relations. He gave the following example of terminating⁸ a legal relation:⁹

. . . X, the owner of ordinary personal property "in a tangible object" has the power to extinguish his own legal interest (rights, powers, immunities, etc.) through that totality of operative facts known as abandonment; and — simultaneously and correlatively — to create in other persons privileges and powers relating to the abandoned object, — e.g., the power to acquire title to the latter by appropriating it. Similarly, X has the power to transfer his interest to Y, — that is, to extinguish his own interest and concomitantly create in Y a new and corresponding interest.

Hence, Hohfeld's treatment of 'power' is significantly different from the treatment of 'right' and 'privilege'. The latter pair of terms refer to three-term relationships, while 'power' is, in effect, stipulated to refer to a two-term relationship between (a) the changing of a legal relation and (b) the power-holder. To say that x has the (legal) power to create legal relation r is the same as to say that legal relation r is not so now, that it is within the volitional control of x to do p, and that if x does p then legal relation r will be created.

Right₄. Hohfeld saw parallels between the right-privilege relationship and the power-immunity relationship. In his words:

... A right is one's affirmative claim against another, and a privilege is one's freedom from the right or claim of another. Similarly, a power is one's affirmative "control" over a given legal relation as against another; whereas an immunity is one's freedom from the legal power of "control" of another as regards some legal relation. ... X, a landowner, has, as we have seen, power to alienate to Y or to any other ordinary party. On the other hand, X has also various immunities as against Y, and all other ordinary parties. For Y is under a disability (i.e., has no power) so far as shifting the legal interest either to himself or to a third party is concerned; and what is true of Y applies similarly to every one else who has not by virtue of special operative facts acquired a power to alienate X's property.

Like the treatment of 'power', the term 'immunity' is, in effect, treated as referring to a two-term relationship between (a) the changing of a legal relation and (b) the disability-holder (the person who does not have power to change that legal relation). To Hohfeld, to say that x has an immunity from y's control with respect to creating legal relation r would be to say (a) that legal relation r is not so now and (b) that although it may be within the volitional control of y to do p, it is not so that by virtue of y's doing p that legal relation r will be created. If the concepts of power and immunity were going to be treated in detail here (they are not), it would be useful to amend this slightly by saying that to say that legal relation r has an immunity from y's control would be to say that (a) legal relation r is not so now and (b) that although it may be within the volitional control of y to do p, it is not so that by virtue of y's doing p that legal relation r will be created.

The remarks above about Rights_(1,2,3,& 4) are slight elaborations on what Hohfeld actually said about 'right', 'privilege', 'power', and 'immunity', but they are only clarifications that facilitate transition to the discussion of a formally defined concept of right to be pursued here and are not in any way inconsistent to Hohfeld's ideas. It is the idea of right (in the strict sense), indicated by Hohfeld by the term 'right' and indicated here by the term 'Right₁', that attention shall be focused on. Some considerations involved in formally defining the concept of right will be explored, and a preliminary proposal for defining it will be undertaken. The relationship of the proposed defined concept of right to proposed defined concepts of *privilege*, *duty*, and *noright* will be considered, and some of the relationships of right to proposals that will be made in the future for defining *power*, *immunity*, *liability*, and *disability* will be briefly sketched.

Hohfeld's monumental contribution in clarifying the language available for discussing law was the precursor of legal realism.^{9a} The magnitude of his influence is revealed in the efforts of his disciples — Walter Wheeler Cook, Arthur L. Corbin, and Karl Llewellyn — who

were among the leading lights of realism in law. Further refinement along the lines that Hohfeld charted may well pave the way for another major breakthrough in legal thought and practice by significantly enhancing the compatibility of law and computers and the usefulness of the latter to the former in the emerging electronic age. It is to this purpose that these preliminary efforts toward formalizing the Hohfeldian system of legal analysis are addressed.

Before turning to the formal definition of right, there is a preliminary matter to be considered. It would be useful to have a clear and unambiguous way of indicating the occurrence of a defined term. A commonly-used method in legal writing is to capitalize the first letter of the defined term. Hence, by stipulation a Defined term (like this one, i.e., the word 'Defined') is a word whose initial letter is capitalized. This works fine most — but not all — of the time. Defined terms may appear as the first words of a sentence. Then it is not clear whether the word is

- a) being used in its Defined sense and therefore being capitalized for being both the initial word of a sentence and occurrence of a word used in a Defined sense, or
- b) being used in a sense other than its Defined sense and therefore being capitalized only because it is the first word of a sentence.

An alternative method for indicating a Defined term is to capitalize both the first and last letters of the term. Defined terms used in their Defined sense would then be distinguished from those same terms used in a sense other than their Defined sense. This is the method that will be used for indicating occurrences of Defined terms in this article. Thus, 'RighT' will indicate an occurrence of a Defined term.

Formal Definition of 'RighT'^{9b}

In approaching the task of formally defining the term 'Right', recall that to say (in a Hohfeldian sense) that x has a (legal) right that p shall be done by y is the same as to say that it is obligatory that p be done by y for x and that the legal system will enforce the obligation. The statement 'x has a right that p shall be done by y' can be abbreviated by the expression 'Right-pyx'. Hereafter, such abbreviations will be indicated by statements such as the following:

- (1) RighT-pyx =_{ab} x has a right that p shall be done by y.

The '=_{ab}' indicates that the statement on its right is equal (by abbreviation) to the statement on its left.

Note that to say that it is obligatory that *p* be done by *y* for *x* and that the legal system will enforce the obligation is the same as to say that it is (legally) obligatory that *p* be done by *y* for *x*. The various parts of the statement 'it is (legally) obligatory that *p* be done by *y* for *x*' can be abbreviated as follows:

- (2) 0 =ab it is (legally) obligatory that
- (3) D2py =ab *p* is done by *y*
- (4) D4px =ab *p* is done for *x*
- (5) D24pyx =ab *p* is done by *y* for *x*.

Using these abbreviations the formal contextual definition of 'RighT' is as follows:

- (6) RighT-pyx =df OD24pyx.

It may be helpful in reading an expression like 'RighT-pyx' to consider its similarity to another statement that contains a three-term relation. Consider the three-term relation indicated by the term 'Between' in a statement like 'Philadelphia is between Boston and Washington'. The statement can be abbreviated as follows:

- (7) Between-bwp =ab Philadelphia is between Boston and Washington.

This abbreviation indicates that 'Between' is a three-term relation that relates the terms 'b', 'w', and 'p' just as 'RighT' is a three-term relation that relates the terms 'p', 'y', and 'x'. Another way of looking at 'Between-bwp' is to regard 'Between-bw' as a property (or predicate or one-term relation) of *p* so that 'Between-bwp' is regarded as saying that Philadelphia (*p*) has the property (Between-bw) of being between Baltimore and Washington. Similarly, instead of regarding 'RighT' as a three-term relation in 'RighT-pyx', we could just as well look at 'RighT-py' as a property of *x* so that 'RighT-pyx' would be interpreted as saying that *x* has the property (RighT-py) of having a right that *p* be done by *y*. Hence, just as:

- (8) Rb =ab The ball has the property of being round
(which is another way of saying that the ball is round).

is used in standard logical notation to indicate the property (roundness) is predicated of the object (the ball), so also:

- (9) RighT-pyx =ab *x* has the property of having a RighT-py.

This may be helpful in appreciating the rationale underlying the (seemingly) peculiar order of the symbols in 'RighT-pyx' in which

the right-holder, x , appears last even though in the statement 'x has a right that p be done by y ' x appears first.

Since 'Right' in (6) is defined in terms of ' \circ ' and 'D24', we shall have a formal definition of 'Right' only when ' \circ ' and 'D24' have been formally defined. First we shall consider the formal definition of ' \circ ', which is used to indicate the concept of legal obligation. We shall, in the manner of Alan R. Anderson,¹⁰ treat ' \circ ' as a unary operator operating on a proposition (the idea expressed by a sentence), rather than as operating on a class (of acts) in the manner of Georg von Wright.¹¹

There are certain properties that any adequate definition of legal obligation should have. The six properties that will be considered here are not in any sense thought to be a comprehensive list. They happen to be six that distinguish among various formal systems that have been formulated by logicians as possible candidates to be used for defining the concept of obligation. The concept of legal obligation that is defined in a formal system should (I believe, at this time) be such that it has the following six properties:

- (P1) It is not provable in the formal system that from 'it is (legally) obligatory that p ', it is valid to infer 'it is (legally) obligatory that it is (legally) obligatory that p '.

Using ' $\neg\circ$ ' to indicate non-provability, (P1) can be abbreviated as follows:

- (P2) $Op \neg\circ OOp$.
(It is not provable in the formal system that from 'it is (legally) obligatory that p ', it is valid to infer 'it is logically necessary (L) that it is (legally) obligatory that p '.)
- (P3) $\neg\circ O(\text{if } p, \text{ then } p)$.
(It is not provable in the formal system that logical tautologies (such as, 'if p , then p ') are (legally) obligatory.)
- (P4) $p \neg\circ OPp$.
(It is not provable in the formal system that from ' p ', it is valid to infer 'it is (legally) obligatory that it is (legally) permitted that p ' (where ' P ' indicates legal permission and ' Pp ' is defined as 'not obligatory not p ').)
- (P5) It is provable in the formal system that 'it is (legally) obligatory that if it is (legally) obligatory that p , then p is true'. (In other words, legal obligations (legally) ought to be fulfilled.)

Using '—**' to indicate provability, (P5) can be abbreviated as follows:

—** O (if Op, then p).

(P6) Op, O (if p, then q) —** Oq.

It is provable in the formal system that from 'it is obligatory that p' and 'it is (legally) obligatory that if p, then q', it is valid to infer 'it is (legally) obligatory that q'.

For the first four properties, it is evident and easy to illustrate that any reasonable formal definition of the concept of legal obligation should result in the non-provability statements (P1)-(P4). If instead of (P1), Op —** OOp, then from a statement such as:

(10) It is obligatory in Michigan now that persons with earnings of \$10,000 file a state income tax statement.

it would be possible to prove as a matter of logic:

(11) it is obligatory that (10).

Certainly, just because (10) happens to be true, we would not want the concept of legal obligation so defined that as a matter of logic it is provable that (11) is true—in effect, that it is (legally) obligatory that Michigan have a state income tax. Although it is certainly true that it is (legally) permitted that Michigan have a state income tax, it is just as true that it is also (legally) permitted that Michigan not have a state income tax. In fact, in 1960 it did not have such a tax, while in 1968 it did—and both were (legally) permitted. Furthermore, if the Michigan legislature wishes to terminate this tax, it certainly has the legal power to do so. Hence, just because (10) happens to be true, we would not want to define the concept of legal obligation in such a way that it is, therefore, legally obligatory for (10) to be true.

The argument in favor of (P2), rather than OP —** LOP, is similar. Just because (10) happens to be true, we would not want to so define the concept of legal obligation that it can be logically proved that

(12) It is logically necessary that (10).

If it were necessary as a matter of logic that (10) be true, then it would be, in effect, logically impossible for Michigan not to have a state income tax. That is clearly absurd. As a matter of fact, until very recently Michigan has not had such a tax.

To argue the concept of legal obligation should be so defined that —** O (if p, then p), rather than (P3), is the same as asserting that for (if p, then p) not to be so is a violation of the law. There is not a single statute, constitutional provision, regulation, or other (legal) normative command that (to my knowledge) so provides. If tautologies were universally obligatory (legally) as a matter of logic, then there certainly should be more indications of it in (legal) normative commands. As a matter of fact, it would seem to be the extreme of

redundancy for legislatures to spend their time commanding that logical truths be so. If a proposition about the state of affairs is, in fact, logically true, then it would be impossible for it to be otherwise. Such an obligation would be one that would be impossible to violate. And what kind of obligation would that be! Certainly it would not be one that would likely have much effect upon human behavior, if that is what legal norms are intended to do. It seems clearly more desirable to so define the concept of legal obligation that (P3) is the case, that is, so that it is not (logically) provable that tautologies are (legally) obligatory.

Finally, with respect to the fourth non-provable statement involving the concept of legal obligation — namely, (P4) — it would be strange beyond belief for obligation (of any kind, legal included) to be defined otherwise. To contend that $p \rightarrow ** O p$ is to contend that it is against the law for anything that is, in fact, done to be forbidden by the law. This would forbid the legislature (as a matter of logic) from commanding or prohibiting any kind of human behavior. To deny (P4) would just be too bizarre to consider seriously.

The final pair of properties that should be among the minimum prerequisites of any adequate definition of legal obligation involve statements of provability about statements involving legal obligation. Unlike the first four, these cannot be justified by merely giving counter-examples. To argue for (P5) is to contend that as a matter of logic it should be provable that there is a violation of the law if it is not so that if $O p$ then p . Notice that this contention is quite different from saying that if $O p$ then p . The latter is saying that everything that is obligatory is, in fact, done. Alternatively, to say that if $O p$ then p is to say that there are no violations of law. That is clearly not so, and nobody would seriously so contend. All that ' $O(\text{if } O p, \text{ then } p)$ ' asserts is that ' $(\text{if } O p, \text{ then } p)$ ' should, as a matter of law, be so; it does not assert that it is, in fact, so. Another way of putting it would be to say that the concept of legal obligation should be so defined that if there is no violation of the legal command that legal norms should be fulfilled, then the legal norms, in fact, have been fulfilled. It would be strange to define legal obligation so that this would be otherwise. To do so would be to accept the possibility that there could be a violation of some legal obligation, but this violation would not be a violation of an obligation that legal obligations be fulfilled.

Finally, to argue for ' $O p, O(\text{if } p, \text{ then } q) \rightarrow O q$ ', rather than (P6) is to maintain that it should be logically possible for there to be an obligation that p and an obligation that if p then q , and despite this pair of obligations, for ' q ' not to be so without violating the legal system. That this is (logically) unreasonable is apparent from a consideration of the two possible situations: (a) p and not q , and (b) not p and not q . These are the only possible situations where ' q ' is not so, and both clearly lead to a violation. In situation (a), it must be the case that ' $\text{not } (\text{if } p \text{ then } q)$ ', because ' $\text{not } q$ ' follows from ' p ' and

'if p then q'. Therefore, the second obligation would be violated, and hence, there would be a violation of one of the norms of the legal system, namely, 'O(if p then q)'. In situation (b), the first obligation, namely, 'Op', is violated, and hence, there is a violation of one of the norms of the legal system. Since in all cases, 'not q' leads to a violation of the legal system, by definition, it is obligatory that q.

By means of these six criteria we shall be able to distinguish from eight different formal systems of logic the one that leads to the most adequate definition of legal obligation in the sense that it has all six properties. But before considering the first formal system, we should examine in closer detail what is meant by the expression 'it is (legally) obligatory that'. It can be contextually defined as follows:

(13) Op =df If not p, then there is a violation.

Thus, in some of the systems that will be considered here, 'It is obligatory that' is defined in terms of 'if-then', 'not', and 'violation', which in turn will be defined in the formal system. The concept of violation is defined in terms of the particular individual violations of particular legal norms of the legal system.

(14) V =ab There is a violation.

It will be the concept that links the formal system realistically to the legal system. Operationally in real-world experience when there is a particular individual violation, if the matter is appropriately brought to the attention of the authorized community decision makers, they will bring the resources of the community to bear in pressuring the violator. It should also be noted that 'V', in indicating a violation in the sense of a violation of the legal system of norms, rather than the violation of a particular norm, is exactly like Anderson's 'S' (disjunction of all sanctions); 'V' is the disjunction of all particular violations ($V_1, V_2, \dots V_n$).

Logical System LS1

The first logical system to be considered as a possible candidate for use in defining 'O' is the ordinary two-valued propositional logic, here called LS1. LS1 is formulated in the subordinate-proof style of Frederic Fitch¹² and in the parenthesis-free notation of Jan Lukasiewicz,¹³ as are the other seven systems to be considered. The 'if-then' formalized in LS1 is material implication and is represented by the connective 'C'. Negation is represented by the connective 'N'. Using these, 'It is obligatory that' (hereafter abbreviated 'Obligation') is defined in the following contextual definition:

(15) $Ow =df CNwV.$

(‘It is obligatory thaT w’ is equal to by definition ‘If not w, then there is a violation’.)

LS1 can be formulated by the following alphabet, formation rules, transformation rules, and definitions:

Alphabet

Variables	$p\ q\ r\ s\ s_5\ s_6\ \dots$
Connectives	$C\ K\ N$
Meta-Variables	
Formulas	$e\ f\ f_3\ f_4\ \dots$
WFFs (well-formed formulas)	$u\ v\ w\ w_4\ w_5\ \dots$

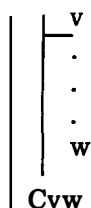
Formation Rules

- FR1 If a formula is a variable, then it is a WFF.
- FR2 If formulas e and f are WFFs, then
 (a) so are Kef and Cef , and
 (b) so is Nf .
- FR3 If a formula is not a WFF by virtue of one of the
 above rules, then it is not a WFF.

Transformation Rules

	<i>Name of Rule</i>	<i>Statement of Rule</i>
TR1	Ko:	$Kvw \multimap v, w.$ (From the K-WFF ‘ Kvw ’, it is assumed to be valid to infer the WFF ‘ v ’ and to be valid to infer the WFF ‘ w ’.)
TR2	Ki:	$v, w \multimap Kvw.$ (From ‘ v ’, ‘ w ’, it is assumed to be valid to infer ‘ Kvw ’.)
TR3	Co:	$Cvw, v \multimap w.$ (From ‘ Cvw ’ and ‘ v ’, it is assumed to be valid to infer ‘ w ’.)
TR4	Ci:	$(v \multimap w) \multimap Cvw.$ (From the provability, in a subordinate proof, of ‘ w ’, given that ‘ v ’ is assumed to be true, it is assumed to be valid to infer ‘ Cvw ’. In the statement of

the Ci Rule, the parentheses are used to indicate a subordinate proof. In the vertical style used by Fitch, this would be written:



- TR5 R: $w \rightarrow (v: \dots w)$.
 (From 'w', it is assumed to be valid to infer 'w' in a subordinate proof in which 'v' is a supposition.)
- TR6 No: $(Nv \rightarrow ** w, Nw) \rightarrow v$.
 (From the provability in a subordinate proof of 'w' and 'Nw', given that 'Nv' is assumed to be true, it is assumed to be valid to infer 'v'.)

Definitions

- D1 Ad: $Avw =df NKvNw$.
 ('v or w' is equal to by definition 'it is not so that (not v and not w)').
- D2 Ed: $Evw =df KCvwCwv$.
 ('v is equivalent to w' is equal to by definition 'if v then w, and if w then v').

In LS1 so formulated, it is provable that 'C-NCNpV-V' can be inferred from 'CNpV'. Since 'Ow' is 'CNwV', the following is the case: $Op \rightarrow ** OOp$. Hence, (P1) is not fulfilled in LS1. Similarly, it can be shown that in LS1 (P3) is not fulfilled (because $\rightarrow ** C-NCpp-V$, and hence, $\rightarrow ** OCpp$) and also that (P4) is not fulfilled (because $p \rightarrow ** C-C-NCNNpV-V$, and hence, $p \rightarrow ** OOp$). This leads to the conclusion that 'O' as defined is not adequately formalized in LS1 because the definition of 'O' in this system does have the first, third, and fourth properties that an adequate definition of legal obligation should, as a minimum, have. The concept of obligation defined in LS1 would, however, be adequate with respect to (P2), (P5), and (P6).

Logical System LS2

The efforts by logicians to formalize more adequately the concept of 'if-then' have led to consideration of the systems of logic, sometimes referred to as "modal" logic (or more precisely as "alethic"

logic). Here we shall consider the possibilities of defining 'O' in two of the standard systems of alethic logic — the one called System M and the one called System S4. The 'if-then' concept of Systems M and S4 is usually called "strict" or "logically necessary" implication in contrast to the "material" implication of LS1. In these two systems, the concept of "logical necessity" is here indicated by the symbol 'L'. The expression 'LCpq' will be read as 'p necessarily implies q' and will represent a different sense of 'if p then q' than is represented by 'Cpq' in LS1.

In System M, it is possible to prove everything that is provable in LS1; in addition, some further theorems can be proved. When two systems are related in this way, the system in which more theorems are provable is called an "extension" of the other system. Hence, M is an extension of LS1. There is a sense in which LS1 is included in M and a sense in which M can be built upon the set of assumptions that constitute the formulation of LS1. Here M will be formulated as those assumptions (alphabet, formation rules, transformation rules, and definitions) of LS1 plus some additional ones. The additional ones are as follows:

Alphabet

Constants	$V_1 V_2 V_3 V_4 \dots$
Connectives	L

Formation Rules

- FR1 If a formula is a variable or a constant, then it is a WFF.
 FR2 (c) so is Lf.

Transformation Rules

	<i>Name of Rule</i>	<i>Statement of Rule</i>
TR7	Lo:	$Lw \rightarrow w$. (From 'w is true necessarily as a matter of logic' it is assumed to be valid to infer 'w'.)
TR8	L(o)Li:	$L(\dots w) \rightarrow Lw$. (From an L-restricted subordinate proof that has 'w' as an item, it is assumed to be valid to infer 'Lw'.) For purposes of LS2, an L-restricted subordinate proof is one that only L-WFFs can be reiterated into. Reiteration into L-restricted subordinate proofs is done by means of TR9, which is similar to the R rule of LS1. Both are referred to as reiteration rules

—TR5 for reiterating into unrestricted subordinate proofs and TR9 for reiterating into L-restricted subordinate proofs.

- TR9 LoL(j): $Lw \rightarrow L(v: \dots w), L(\dots w).$
 (From 'Lw', it is assumed to be valid to infer 'w' in an L-restricted subordinate proof in which 'v' is a supposition, and it is assumed to be valid to infer 'w' in an L-restricted subordinate proof in which there are no suppositions.)

Definitions

- D3 Md: $Mw \text{ =df } NLNw.$
 ('It is possible that w' is equal to by definition 'it is not so that it is logically necessary that not w'.)

System LS2 is an extension of M obtained by adding the following axiom and definitions to M:

- A1 MNVa: MNV
 (It is logically possible that there is no violation.)
- D4 Vd: $V \text{ =df } \underbrace{AA \dots AV_1}_{n} V_2 \dots V_{n+1}.$
 ('V' is equal to by definition the disjunction of all the violations of a legal system that there can be, that is, if 'V' is true then there is at least one violation of the legal system.)
- D5 Od: $Ow \text{ =df } LC-Nw-V.$
 ('Ow' is equal to by definition 'LC-Nw-V', which when interpreted is: 'it is obligatory that w' is equal to 'if not w, then there is a violation'.)
- D6 Pd: $Pw \text{ =df } NONw.$
 ('Pw' is equal to by definition 'NONw', which when interpreted is: 'it is permitted that w' is equal to 'it is not obligatory that not w'.)

In LS2 so formulated with 'Ow' so defined, it can be ascertained that the following is the case:

- (16) $\rightarrow ** \text{ OLCpp.}$
 (17) $\rightarrow o \text{ OLC-Op-p.}$

Hence, neither (P3) nor (P5) is fulfilled in LS2, and thus 'O' is not adequately formalized in this system.

Logical System LS3

System S4 is a system of alethic logic that is an extension of M. It can be formulated by merely replacing TR9 of M by the following

slightly stronger rule for reiterating into L-restricted subordinate proofs:

- TR10 $LoL(L)i:$ $Lw \rightarrow L(v: \dots LW), L(\dots Lw).$
 (From 'Lw' it is assumed to be valid to infer 'Lw' in an L-restricted subordinate proof in which 'v' is a supposition, and it is assumed to be valid to infer 'Lw' in an L-restricted subordinate proof in which there are no suppositions.)

LS3 is an extension of S4, and it is also an extension of LS2. LS3 can be obtained by adding axiom A1 and definitions D4, D5, and D6 to S4; or alternatively, it can be obtained by replacing TR9 in LS2 by TR10. In LS3 so formulated with 'Ow' so defined, it can be ascertained that not only are (16) and (17) the case but also the following:

$$(18) \quad Op \rightarrow ** OOp.$$

$$(19) \quad Op \rightarrow ** LOp.$$

Hence, not only (P3) and (P5) but also (P1) and (P2) fail to be fulfilled in LS3, and thus 'O' is not adequately formalized in this system either.

Logical System LS4

Frederic Fitch has formulated a system that is an extension of S4.¹⁴ Instead of defining 'O' in terms of 'LC' and 'V' (as in LS2 and LS3), he introduces 'O' by means of the following four transformation rules:

	<i>Name of Rule</i>	<i>Statement of Rule</i>
TR11	$O()oOi:$	$O(\dots w) \rightarrow Ow.$ (From an O-restricted subordinate proof that has 'w' as an item, it is assumed to be valid to infer 'Ow'.)
TR12	$O(Oo):$	$O(Ow \rightarrow w).$ (Within an O-restricted subordinate proof, from 'Ow' it is assumed to be valid to infer 'w'.)
TR13	$OoOo:$	$Ov, ONv \rightarrow w.$ (From 'Ov' and 'ONv', it is assumed to be valid to infer 'w'.)
TR14	$OoO(O)i:$	$Ow \rightarrow O(v: \dots Ow), O(\dots Ow).$ (From 'Ow', it is assumed to be valid to infer 'Ow' in an O-restricted subordinate proof in which 'v' is a supposition, and it is assumed to be valid to infer 'Ow' in an O-restricted subordinate proof in which there are no suppositions.)

The result when these four rules are added to S4 is System F(S4) (here called LS4). So formulated, LS4 is just like LS3 with respect to the six properties being considered here of an adequate definition of legal obligation, except that (P2) is fulfilled for LS4 while it is not for LS3.

Logical System LS5

When these four obligation transformation rules (TR11-TR14) are added to System M, the resulting system is System F(M) (here called LS5). LS5 is exactly like LS2 with respect to the six properties being considered: neither (P3) nor (P5) is fulfilled, but the other four are. Clearly, in the five systems considered so far, the troublesome properties are (P3) and (P5). In none of the five systems is (P3) fulfilled, and (P5) is fulfilled only in LS1. Next to be examined is a system that comes to grips with (P3), the most pervasive difficulty.

Logical System LS6

An approach to formulating logical systems developed by Anderson in collaboration with Nuel D. Belnap¹⁵ has led to the desired result with respect to (P3). In one of Anderson's systems there is specified a formalization of 'if-then' that more closely approximates its meaning in English prose, i.e., that the consequent somehow logically follows from and is dependent upon the antecedent. In System E¹⁶ when it is asserted that the following is true:

- (20) if p then q
 (that is, 'p entails q', or in notation 'Tpq')

two conditions are required to be fulfilled that, for example, need not necessarily be fulfilled with respect to 'Cpq' of LS1:

- (21) the truth of 'q' follows from the truth of 'p',
and
(22) the truth of 'q' is dependent upon the truth of 'p'.

What these two requirements of relevance and dependence preclude are the provability in E of such things as 'T-p-Trr' and 'T-p-Trp', whereas in LS1 'C-p-Crr' and 'C-p-Crp' are provable. With respect to 'T-p-Trr', (22) is not fulfilled because the truth of 'Trr' is not dependent upon the truth of 'p'. On the other hand, with respect to 'T-p-Trp' (21) is not fulfilled because if 'Trp' follows from the truth of 'p', then when 'p' is true so is 'Trp'. This, in turn, means that when 'p' is true the two requirements must be fulfilled for the 'r' and 'p' of 'Trp' for 'Trp' to be true. But when 'p' is true for empirical reasons, for example, the truth of 'p' in 'Trp' does not follow from the truth of 'r' (leaving (21) not fulfilled), and it is not dependent upon the truth of 'r' (leaving (22) not fulfilled). Hence, (21) is not always fulfilled for 'T-p-Trp'.

If the 'LC' of LS2, LS3, LS4, and LS5 is interpreted as entailment, the restrictions upon reiteration into L-restricted subordinate proofs constrain the systems so that 'LC-p-LCqp' is not provable; therefore, requirement (21) is fulfilled. However, (22) is not fulfilled in these systems, because 'LC-p-LCrr' is provable. Anderson builds (22) into LS6 by specifying a subscripting notation for keeping track of all the suppositions actually used in deducing and restricting the entailment-in rule—i.e., T(o)Ti—so that only if supposition 'w' is used in deducing 'v', will it be valid to infer that 'Twv' follows from a proof of 'v', given 'w'.¹⁷ Requirement (21) is built into LS6 the same way that it is in LS2, LS3, LS4, and LS5—by restrictions upon reiteration into restricted subordinate proofs.

The set of assumptions for formulating a logical system is called the "basis" of that system. The basis of System E is as follows:

Alphabet

Variables

Sentence	$p \ q \ r \ s \ s_5 \ s_6 \ \dots$
----------	-------------------------------------

Numerical Subscripts

Individual	$[i]$
------------	-------

Set	$a \ b$
-----	---------

Logical Sum	aUb
-------------	-------

Logical Difference	$a-b$
--------------------	-------

Constants	$V_1 \ V_2 \ V_3 \ \dots$
-----------	---------------------------

Connectives	$K \ T \ A \ N$
-------------	-----------------

Meta-Variables

Formulas	$e \ f \ f_3 \ f_4 \ \dots$
----------	-----------------------------

WFFs	$u \ v \ w \ w_4 \ w_5 \ \dots$
------	---------------------------------

Formation Rules

FR1 If a formula is a variable or a constant, then it is a WFF.

FR2 If formulas e and f are WFFs, then

- (a) so are Kef , Tef , and Aef , and
- (b) so are f_a and Nf .

- FR3 If a formula is not a WFF by virtue of one of the above rules, then it is not a WFF.

Transformation Rules

<i>Name of Rule</i>	<i>Statement of Rule</i>
Ko':	$Kvw_a \text{ --- } v_a, w_a.$ <p>where 'a' indicates the set of numerical subscripts on 'Kvw' that is carried along to 'v' and 'w'.</p> <p>(From 'Kvw_a' (i.e., 'v and w'), it is assumed to be valid to infer 'v_a' and to be valid to infer 'w_a'.)</p>
Ki':	$v_a, w_a \text{ --- } Kvw_a.$ <p>where 'a' indicates that the set of numerical subscripts on 'v' and 'w' must be identical and that the same set of subscripts is carried along to 'Kvw'.</p> <p>(From 'v_a' and 'w_a', it is assumed to be valid to infer 'Kvw_a' (i.e., 'v and w').)</p>
To:	$Tvw_a, v_b \text{ --- } w_aUb.$ <p>where 'a' and 'b' indicate that the sets of numerical subscripts on 'Tvw' and 'v' may be different and 'aUb' indicates that the set of subscripts carried along to 'w' is the logical sum of 'a' and 'b'.</p> <p>(From 'Tvw_a' (i.e., 'v entails w') and 'v_b', it is assumed to be valid to infer 'w_aUb'.)</p>
T()oTi:	$T(v[i] \text{ --- } w_a) \text{ --- } Tvw_{a-[i]}.$ <p>where '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition, 'a' is a set of subscripts which contains '[i]', and '$a-[i]$' is a set of subscripts comprised of those in 'a' with '[i]' deleted.</p> <p>(From the provability, in a T-restricted subordinate proof, of 'w_a', given that '$v[i]$' is assumed to be true, it is assumed to be valid to infer '$Tvw_{a-[i]}$' (i.e., 'v entails w').)</p>

ToT(T)i: $\text{Tuv}_a \text{---}^* \text{T}(\text{w}_{[i]} : \dots \text{Tuv}_a),$
 $\text{T}(\dots \text{Tuv}_a).$

where 'a' indicates the set of numerical subscripts on 'Tuv' that is carried along upon reiteration into a T-restricted subordinate proof and '[i]' indicates a numerical subscript assigned to supposition 'w' which is distinct from the numerical subscript assigned to any other supposition.

(From ' Tuv_a ' (i.e., 'u entails v'), it is assumed to be valid in a T-restricted subordinate proof to infer ' Tuv_a ', given that ' $\text{w}_{[i]}$ ' is assumed to be true, and it is assumed to be valid in a T-restricted subordinate proof to infer ' Tuv_a '.)

Rp': $\text{w}_a \text{---}^* \text{w}_a.$

(From ' w_a ', it is assumed to be valid to infer ' w_a '.)

AoNKi': $\text{Avw}_a \text{---}^* \text{NKNvNw}_a.$

(From ' Avw_a ' (i.e., 'v or w'), it is assumed to be valid to infer ' NKNvNw_a ' (i.e., 'not (not v and not w)').)

NKoAi': $\text{NKNvNw}_a \text{---}^* \text{Avw}_a.$

(From ' NKNvNw_a ' (i.e., 'not (not v and now w)'), it is assumed to be valid to infer ' Avw_a ' (i.e., 'v or w').)

KoAi2': $\text{KuAvw}_a \text{---}^* \text{AKuvw}_a.$

(From ' KuAvw_a ' (i.e., 'u and (v or w)'), it is assumed to be valid to infer ' AKuvw_a ' (i.e., '(u and v) or w').)

ToNo: $\text{Tw}_a, \text{Nw}_b \text{---}^* \text{Nv}_a \text{Ub}.$

where 'a' and 'b' indicate that the sets of numerical subscripts on ' Tw ' and ' Nw ' may be different and ' aUb ' indicates that the set of subscripts carried along to ' Nv ' is the logical sum of 'a' and 'b'.

(From ' Tw_a ' (i.e., 'v entails w') and ' Nw_b ' (i.e., 'not w'), it is assumed to be valid to infer ' $\text{Nv}_a \text{Ub}$ ' (i.e., 'not v').)

$T()oNi:$ $T(v[i] \text{ ---} w_a, Nw_b) \text{ ---} Nv_{aUb-[i]}.$

where 'a' and 'b' indicate that the sets of numerical subscripts on 'w' and 'Nw' may be different, '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition and is contained in both 'a' and 'b', and 'aUb-[i]' indicates that the set of subscripts carried along to 'Nv' is comprised of those in 'aUb' with '[i]' deleted.

(From the provability in a T-restricted subordinate proof of 'w_a' and 'Nw_b', given that 'v_[i]' is assumed to be true, it is assumed to be valid to infer 'Nv_{aUb-[i]}'.)

NNi':

$w_a \text{ ---} NNw_a.$

(From 'w_a', it is assumed to be valid to infer 'NNw_a' (i.e., 'not with w').)

NNo':

$NNw_a \text{ ---} w_a.$

(From 'NNw_a' (i.e., 'not not w'), it is assumed to be valid to infer 'w_a'.)

The following tabulated summary of a proof of the transitivity of 'T' (entailment) illustrates a proof in System E.

ToToTi	TPq ₁ ,	Tqr ₂	---	Tpr ₁₂
1	TPq ₁			s
2	Tqr ₂			s
3	Ta	p ₃		s
	b	TPq ₁		1, ToT(T)i
	c	q ₁₃		b, a, To
	d	Tqr ₂		2, ToT(T)i
	e	r ₁₂₃		d, c, To
4	Tpr ₁₂			3, T()oTi

Several things about the proof of ToToTi should be noted:

- (a) Each supposition is assigned a unique numerical subscript (items 1, 2, and 3a).

- (b) When 'q' is inferred from 'Tpq' and 'p' as item 3c, their subscripts, 1 and 3, respectively, are carried along to 'q'. Similarly, for item 3e.
- (c) When 'Tpr' is inferred as item 4 from the proof of 'r', given 'p' as a supposition, the 3-subscript of 'p' is contained in the 123-subscript of 'r', and the 12-subscript of 'Tpr' is the result of deleting 3 from 123.

A typographically more convenient as well as more perspicuous summary tabulation for checking purposes results if subscripts are elevated to the line-level of the WFF and listed in a column between the proof and its justification, as in the following tabulation of a proof:

—** T-TpTqr-TKpqr						
1	Ta	TpTqr		1	s	
		b	T1	Kpq		2
	2			TpTqr		1
	3		p		2	1,Ko
	4		Tqr		12	2,3,To
	5		q		2	1,Ko
	6		r		12	4,5,To
	c		TKpqr		1	b,T()oTi
	2	T-TpTqr-TKpqr				1,T()oTi

The 'if-then' (represented by 'T') formalized in System E thus formulated permits formalization of a concept of legal obligation that does fulfill (P3). The following set of transformation rules, axioms, and definitions, along with those for E, form the basis for LS6, within which 'O' is defined.

$L()oLi'$: $L(\dots w_a) \text{—}^* Lw_a$.

(From an L-restricted subordinate proof that has 'w_a' as an item that is not a supposition, it is assumed to be valid to infer 'Lw_a' (i.e., 'it is logically necessary that w').)

$LoL(L)i'$: $Lw_a \text{—}^* L(v[i]: \dots Lw_a), L(\dots Lw_a)$.

where 'a' indicates the set of numerical subscripts on 'Lw' that is carried along upon reiteration into an L-restricted subordinate proof and '[i]' indicates a nu-

merical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition.

(From 'Lw_a' (i.e., 'it is logically necessary that w'), it is assumed to be valid in an L-restricted subordinate proof to infer 'Lw_a', given that 'v_[i]' is assumed to be true, and it is assumed to be valid in an L-restricted subordinate proof to infer 'Lw_a').)

Lo': $Lw_a \multimap w_a$.

(From 'Lw_a' (i.e., 'it is logically necessary that w'), it is assumed to be valid to infer 'Lw_a').)

Axioms

MNVa: MNV.

(It is logically possible that there is no violation.)

Definitions

Vd: $V =_{df} \underbrace{AA \dots A}_n V_1 V_2 \dots V_{n+1}$.

('There is a violation' is equal to by definition 'there is a violation of particular legal norm #1 or there is a violation of particular legal norm #2, . . ., or there is a violation of particular legal norm # (n+1)', where there are just n+1 norms in the legal system.

Md: $Mw =_{df} NLNw$.

('It is logically possible that w' is equal to by definition 'it is not so that it is logically necessary that not w'.)

Od: $Ow =_{df} TNwV$.

('It is obligatory that w' is equal to by definition 'not w entails there is a violation'.)

Pd: $Pw =_{df} NONw$.

('It is permitted that w' is equal to by definition 'it is not so that it is obligatory that not w'.)

The concept of if-then formalized by 'T' in LS6, when used to relate the forbidden act to the violation in the definition of legal obligation, leads to the following:

(23) \multimap OTpp.

This means that (P3) is fulfilled for the 'O' of LS6. Unhappily, however, the following are also the case:

(24) $Op \rightarrow ** LOp$.

(25) $\rightarrow o OTOpp$.

These mean that (P2) and (P5) are not fulfilled in LS6. Happily, however, both of these can be remedied if 'O' is defined in terms of a still different concept of if-then, which is explored in the next logical system to be considered here.

Logical System LS7

The entailment concept of if-then formalized in LS6 requires both relevance and dependence. In LS7 there is introduced a *weak implication* ('W') concept of if-then, which has the same relevance requirement as entailment, but a slightly weaker dependence requirement. If legal obligation is defined in terms of a weak implication relation between the forbidden state of affairs and the violation, the non-fulfillment of (P2) and (P5) are remedied, but another problem results.

The basis for LS7 can be obtained by making the following changes in LS6:

1. Replace the 'T' in the alphabet by 'W'.
2. Replace the 'Tef' in FR2 by 'Wef'.
3. Replace the transformation rules:

$To, T()oTi, ToT(T)i, ToNo, \text{ and } T()oNi$

by the transformation rules:

$Wo, W()oWi, W(i), WoNo, \text{ and } W()oNI$

shown below.

$Wo: \quad Wvwa \vee wb \rightarrow ** waUb.$

(From 'Wvwa' (i.e., 'v weakly implies w') and 'wb', it is assumed to be valid to infer 'waUb'.)

$W()oWi: \quad W(v[i] \rightarrow ** wa) \rightarrow ** Wvwa-[i]$
where [i] is in a.

(From the provability, in a W-restricted subordinate proof, of 'wa', given that 'v[i]' is assumed to be true, it is assumed to be valid to infer 'Wvwa-[i]' (i.e., 'v weakly implies w').)

$W(i): \quad wa \rightarrow ** W(v[i]: \dots wa), W(\dots wa).$

(From 'wa', it is assumed to be valid in a W-restricted subordinate proof to infer 'wa', given that 'v[i]' is assumed to be true, and it is assumed to be valid in a W-restricted subordinate proof to infer 'wa'.)

WoNo $Wvw_a, Nwb \rightarrow Nv_aUb$.

(From 'Wvw_a' (i.e., 'v weakly implies w') and 'Nwb', it is assumed to be valid to infer 'Nv_aUb'.

W()oNi: $W(v[i] \rightarrow ** w_a, Nwb) \rightarrow Nv_aUb-[i]$

where [i] is in both a and b.

(From the provability in a W-restricted subordinate proof of 'w_a' and 'Nwb', given that 'Nv_[i]' is assumed to be true, it is assumed to be valid to infer 'Nv_aUb-[i]'.)

4. Replace Od by the Od shown below.

Od: $Ow =df WNwV$

('It is obligatory that w' is equal to by definition 'not w weakly implies there is a violation'.)

The relationship between entailment ('T' of LS6) and weak implication ('W' of LS7) can be made more evident by adding the following definition to LS7:

Td: $Tvw =df LWvw$.

('v entails w' is equal to by definition 'it is logically necessary (in the S4 sense) that v weakly implies w'.)

The entailment concept of LS7, thus defined, is exactly the same concept of if-then as the entailment concept of LS6. It is of some interest that 'W' is related to 'T' in the way that 'C' is related to 'LC' (of S4):

$Tvw_a \rightarrow ** Wvw$. $L(\dots Wvw_a) \rightarrow ** Twv_a$.
 $LCvw \rightarrow ** Cvw$. $L(\dots Cvw) \rightarrow ** LCvw$.

In LS7 the concept of legal obligation leads to the following happy results:

(26) $Op \rightarrow o LOp$,

(27) $\rightarrow ** OWOpp$.

Therefore, (P2) and (P5) are fulfilled. However,

(28) $p \rightarrow ** OPp$.

This means that (P4) is not fulfilled. It is, however, the only one of the six requisite properties that the legal obligation concept of LS7 fails to have. In LS8 that last flaw is eliminated.

Logical System LS8

In each of the seven systems considered so far as possible candidates for use in defining the concept of obligation there has been at

least one flaw in terms of the six criteria being used to evaluate the adequacy of proposed definitions of obligation. This is summarized in Table 1 below where the asterisks (*) indicate the unsatisfactory properties of the definition of obligation in each of the seven systems.

Table 1

	LS1	LS2	LS3	LS4	LS5	LS6	LS7
P1	*		*	*			
P2			*			*	
P3	*	*	*	*	*		
P4	*						*
P5		*	*	*	*	*	
P6							

* = definition of obligation is
unsatisfactory in this respect

One system that satisfactorily fulfills the six criteria being used possesses all the complexity of LS7—and then some more. This system, LS8, defines obligation in terms of still another concept of if-then, namely what here shall be called 'natural implication'. *Natural implication*, in turn, is defined in terms of *natural necessity* and *genuine implication*, while *natural necessity* is defined in terms of *the laws of nature* and *genuine implication*. *Genuine implication* is a variant of if-then that is slightly weaker than weak implication. It is like weak implication in every respect except that double negation introduction holds for some but not all of the expressions in the system that defines genuine implication.

The basis of Logical System LS8 is as follows:

Alphabet

Variables

Sentences	p q r s s ₅ s ₆ . . .
Numerical Subscripts	
Individual	[i]
Set	a b
Logical Sum	aUb
Logical Difference	a-b
Constants	Z V ₁ V ₂ V ₃ . . .
Connectives	K G A N B R L M

Meta-Variables	$e f f_3 f_4 \dots$
WFFs	$u v w w_4 w_5 \dots$

Formation Rules

- FR1 If a formula is a variable or a constant, then it is a WFF.
- FR2 If formulas e and f are WFFs, then
- (a) so are Kef , Gef , and Aef , and
- (b) so are f_a , Bf , Rf , Lf , Mf , and Nf .
- FR3 If a formula is not a WFF by virtue of one of the above rules, then it is not a WFF.

Transformation Rules

<i>Name of Rule</i>	<i>Statement of Rule</i>
---------------------	--------------------------

- | | |
|-----------------------|--|
| Ko': | $Kvw_a \text{ --- } * v_a, w_a.$
(From ' Kvw_a ' (i.e., ' v and w '), it is assumed to be valid to infer ' v_a ' and it is assumed to be valid to infer ' w_a '.) |
| Ki': | $v_a, w_a \text{ --- } * Kvw_a.$
where ' a ' indicates that the set of numerical subscripts on ' v ' and ' w ' must be identical and that the same set of subscripts is carried along to ' Kvw '.
(From ' v_a ' and ' w_a ', it is assumed to be valid to infer ' Kvw_a ' (i.e., ' v and w ').) |
| Go': | $Gvw_a, v_b \text{ --- } * w_a U b.$
where ' a ' and ' b ' indicate that the sets of numerical subscripts on ' Gvw ' and ' v ' may be different and ' aUb ' indicates that the set of subscripts carried along to ' w ' is the logical sum of ' a ' and ' b '.
(From ' Gvw_a ' (i.e., ' v genuinely implies w ') and ' v_b ', it is assumed to be valid to infer ' $w_a U b$ '.) |
| G() _o Gi': | $G(v[i] \text{ --- } ** w_a) \text{ --- } * Gvw_a\text{--}[i].$
where ' $[i]$ ' indicates a numerical subscript assigned to supposition ' v ' which is distinct from the nu- |

merical subscript assigned to any other supposition, 'a' is a set of subscripts which contains '[i]', and 'a-[i]' is a set of subscripts comprised of those in 'a' with '[i]' deleted.

(From the provability, in a G-restricted subordinate proof, of ' w_a ', given that ' $v_{[i]}$ ' is assumed to be true, it is assumed to be valid to infer ' $Gvw_{a-[i]}$ ' (i.e., ' v genuinely implies w ').)

G(i)': $w_a \multimap G(v_{[i]} : \dots w_a), G(\dots w_a).$

where 'a' indicates the set of numerical subscripts on 'w' that is carried along on reiteration into a G-restricted subordinate proof and '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition.

(From ' w_a ', it is assumed to be valid in a G-restricted subordinate proof to infer ' w_a ', given that ' $v_{[i]}$ ' is assumed to be true, and it is assumed to be valid in a G-restricted subordinate proof to infer ' w_a '.)

GoNo': $Gvw_a, Nw_b \multimap Nv_{aUb}.$

where 'a' and 'b' indicate that the sets of numerical subscripts on 'Gvw' and 'Nw' may be different and ' aUb ' indicates that the set of subscripts carried along to 'Nv' is the logical sum of 'a' and 'b'.

(From ' Gvw_a ' (i.e., ' v genuinely implies w ') and 'Nw' (i.e., 'not w '), it is assumed to be valid to infer ' Nv_{aUb} ' (i.e., 'not v ').)

G(oNi)': $G(v_{[i]} \multimap w_a, Nw_b) \multimap Nv_{aUb-[i]}.$

where 'a' and 'b' indicate that the sets of numerical subscripts on 'w' and 'Nw' may be different, '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the nu-

merical subscript assigned to any other supposition and is contained in both 'a' and 'b', and ' $aUb-[i]$ ' indicates that the set of subscripts carried along to 'Nv' is comprised of those in ' aUb ' with '[i]' deleted.

(From the provability in a G-restricted subordinate proof of ' w_a ' and ' Nw_b ', given that ' $Nv_{[i]}$ ' is assumed to be true, it is assumed to be valid to infer ' $Nv_aUb-[i]$ '.)

Rp': The same as in LS6.

AoNKi': The same as in LS6.

NKoAi': The same as in LS6.

KoAi2': The same as in LS6.

NNo': The same as in LS6.

Lo': The same as in LS6.

L() \circ Li': The same as in LS6.

Ai': $w_a \text{ --- } * \text{ Avw}_a, \text{ Awv}_a,$

(From ' w_a ', it is assumed to be valid to infer ' Avw_a ' (i.e., 'v or w') and to be valid to infer ' Awv_a ' (i.e., 'w or v').)

KoNNKi': $\text{Kvw}_a \text{ --- } * \text{ NNKvw}_a$

(From ' Kvw_a ' (i.e., 'v and w'), it is assumed to be valid to infer ' NNKvw_a ' (i.e., 'not not v-and-w').)

LoR(L)i': $\text{Lw}_a \text{ --- } * \text{ R}(v_{[i]}: \dots \text{Lw}_a),$
 $\text{R}(\dots \text{Lw}_a)$

where 'a' indicates the set of numerical subscripts on 'Lw' that is carried along upon reiteration into an R-restricted subordinate proof and '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition.

(From ' Lw_a ' (i.e., 'it is logically necessary that w'), it is assumed to be valid in an R-restricted subordinate proof to infer ' Lw_a ', given that ' $v_{[i]}$ ' is assumed to be true, and it is assumed to be valid in an R-restricted subordinate proof to infer ' Lw_a '.)

LoLNNi': $Lw_a \rightarrow * LNNw_a$.

(From ' Lw_a ' (i.e., 'it is logically necessary that w '), it is assumed to be valid to infer ' $LNNw_a$ ' (i.e., 'it is logically necessary that not w ').)

MoNLNi': $Mw_a \rightarrow * NLNw_a$.

(From ' Mw_a ' (i.e., 'it is logically possible that w '), it is assumed to be valid to infer ' $NLNw_a$ ' (i.e., 'it is not logically necessary that not w ').)

NLNoMi': $NLNw_a \rightarrow * Mw_a$.

(From ' $NLNw_a$ ' (i.e., 'it is not logically necessary that not w '), it is assumed to be valid to infer ' Mw_a ' (i.e., 'it is logically possible that w ').)

GoNBNi': $GZw_a \rightarrow * NBNw_a$.

(From ' GZw_a ' (i.e., 'the laws of nature¹⁸ genuinely imply that w '), it is assumed to be valid to infer ' $NBNw_a$ ' (i.e., 'it is not naturally necessary that not w ').)

MKi': $w_a \rightarrow * MKZw_a$.

(From ' w_a ', it is assumed to be valid to infer ' $MKZw_a$ ' (i.e., 'it is logically possible for both the laws of nature and w to be true').)

MKoNGi': $MKZw_a \rightarrow * NGZNw_a$.

(From ' $MKZw_a$ ' (i.e., 'it is logically possible for both the laws of nature and w to be true'), it is assumed to be valid to infer ' $NGZNw_a$ ' (i.e., 'it is not so that the laws of nature genuinely imply that not w ').)

RoGi': $Rw_a \rightarrow * GZw_a$.

(From ' Rw_a ' (i.e., 'it is naturally necessary that w '), it is assumed to be valid to infer ' GZw_a ' (i.e., 'the laws of nature genuinely imply that w ').)

GoRi': $GZw_a \rightarrow * Rw_a$.

(From ' GZw_a ' (i.e., 'the laws of nature genuinely imply that w '), it is assumed to be valid to infer ' Rw_a ' (i.e., 'it is naturally necessary that w ').)

R(o)Ri': $R(\dots w_a) \rightarrow * Rw_a$.

(From an R-restricted subordinate proof that has ' w_a ' as an item that is not a supposition, it is assumed to be valid to infer ' Rw_a ' (i.e., 'it is naturally necessary that w ').)

RoR(R)i': $Rw_a \rightarrow R(v[i] : \dots Rw_a),$
 $R(\dots Rw_a).$

where 'a' indicates the set of numerical subscripts on 'Rw' that is carried along upon reiteration into an R-restricted subordinate proof and '[i]' indicates a numerical subscript assigned to supposition 'v' which is distinct from the numerical subscript assigned to any other supposition.

(From ' Rw_a ' (i.e., 'it is naturally necessary that w'), it is assumed to be valid in an R-restricted subordinate proof to infer ' Rw_a ', given that ' $v[i]$ ' is assumed to be true, and it is assumed to be valid in an R-restricted subordinate proof to infer ' Rw_a '.)

BoMKi': $Bw_a \rightarrow MKZw_a.$

(From ' Bw_a ' (i.e., 'it is naturally possible that w'), it is assumed to be valid to infer ' $MKZw_a$ ' (i.e., 'it is logically possible that both the laws of nature and w are true').)

MKoBi': $MKZw_a \rightarrow Bw_a.$

(From ' $MKZw_a$ ' (i.e., 'it is logically possible that both the laws of nature and w are true'), it is assumed to be valid to infer ' Bw_a ' (i.e., 'it is naturally possible that w').)

Axioms

MNVa: $MNV.$

(It is logically possible that there is no violation.)

Za: $Z.$

(The laws of nature are true.)

Definitions

Vd: The same as in LS6.

Id: $Ivw =df RGv.$

('v naturally implies w' is equal to by definition 'it is naturally necessary that v genuinely implies w'.)

Od: $Ow =df INwV.$

(‘It is obligatory that w’ is equal to by definition
 ‘not w naturally implies that there is a violation’.)

Pd: The same as in LS6.

The relationships between the logical concepts and natural concepts considered with respect to which can and cannot be inferred from each other as formulated in LS8 are summarized in Figure 1.

Figure 1

Lw_a **——** $NMNw_a$

O
,
,
,
,
,
*

Rw_a *——* GZw_a **——* $NBNw_a$ **——** $Iiwww_a$

O
,
,
,
,
,
*
*

w_a

O O
,
,
,
,
,
*
*

Bw_a *——* $MKZw_a$ **——* $NGZw_a$ **——** $NRNw_a$

O
,
,
,
,
,
*
*

Mw_a *——* $NLNw_a$

where —** = provability of validity
 —o = non-provability of validity
 —* = assumption of validity

L = logical necessity
 N = negation
 M = logical possibility
 R = natural necessity
 B = natural possibility
 G = genuine implication
 Z = laws of nature
 I = natural implication
 K = conjunction

The following are also the case in LS8:

$p \rightarrow OOp$
 $Op \rightarrow LOp$
 $\rightarrow OIpp$
 $p \rightarrow OPp$
 $\rightarrow ** OIOpp$
 $Op, OIpq \rightarrow ** Oq$

Hence, all six of the criteria being used to test the adequacy of a definition of the concept of obligation are met by 'O' as defined in LS8. With the complex task of adequately defining obligation now taken care of, there is just one more brief matter to be considered before turning to the formal definition of Right—namely, what it means for something to “be done”, to “be done by someone”, and to “be done for someone”.

Done, Done By, and Done For

To say that something has been *done* is an abbreviated way of making a statement of fact that it is true that a given state of affairs is the case. Similarly, to say that something has been *done by* person *x* is an abbreviated way of stating that responsibility for the fact that a given state of affairs happens to be the case is ascribed to person *x* by virtue of some articulated (or unarticulated) policies. So, too, is saying that something has been *done for* *x* an abbreviated way of stating that *x* is a person on whose behalf a given state of affairs is the case according to some articulated (or unarticulated) policies. Because the formal definition of Right involves such concepts, it will be necessary to add to LS8 provisions for including these “doing” ideas.

Transformation Rules

<i>Name of Rule</i>	<i>Statement of Rule</i>
D _o I _o D _i ':	D _v _a , I _v w _b —* D _w _a U _b . (From 'D _v _a ' (i.e., 'v has been done') and 'I _v w _v ' (i.e., 'v naturally implies w'), it is assumed to be valid to infer 'D _w _a U _b ' (i.e., 'Dw has been done').)
D ₂ oI _o D ₂ i':	D ₂ v _x _a , I _v w _b —* D ₂ w _a U _b . (From 'D ₂ v _x _a ' (i.e., 'v has been done by x') and 'I _v w _b ' (i.e., 'v naturally implies w'), it is assumed to be valid to infer 'D ₂ w _x _a U _b ' (i.e., 'w has been done by x').)
D ₄ oID ₄ i':	D ₄ v _x _a , I _v w _b —* D ₄ w _x _a U _b . (From 'D ₄ v _x _a ' (i.e., 'v has been done for x') and 'I _v w _b ' (i.e., 'v naturally implies w'), it is assumed to be valid to infer 'D ₄ w _x _a U _b ' (i.e., 'w has been done for x').)
D ₂ oD _i ':	D ₂ w _x _a —* D _w _a . (From 'D ₂ w _x _a ' (i.e., 'w has been done for x'), it is assumed to be valid to infer 'D _w _a ' (i.e., 'w has been done').)
D ₄ oD _i ':	D ₄ w _x _a —* D _w _a . (From 'D ₄ w _x _a ' (i.e., 'w has been done for x'), it is assumed to be valid to infer 'D _w _a ' (i.e., 'w has been done').)
OD ₂ oDN _o D ₂ Ni':	OD ₂ w _x _a , DN _w _b —* D ₂ Nw _x _a U _b . (From 'OD ₂ w _x _a ' (i.e., 'it is obligatory that w be done by x') and 'DN _w _b ' (i.e., 'not w has been done'), it is assumed to be valid to infer 'D ₂ Nw _x _a U _b ' (i.e., 'not w has been done by x').)
OD ₄ oDN _o D ₄ Ni':	OD ₄ w _x _a , DN _w _b —* D ₄ Nw _x _a U _b . (From 'OD ₄ w _x _a ' (i.e., 'it is obligatory that w be done for x') and 'DN _w _b ' (i.e., 'not w has been done'), it is assumed to be valid to infer 'D ₄ Nw _x _a U _b ' (i.e., 'not w has been done for x').)
D ₂ NoND ₂ i':	D ₂ Nw _x _a —* ND ₂ w _x _a . (From 'D ₂ Nw _x _a ' (i.e., 'not w has been done by x'), it is assumed to be valid to infer 'ND ₂ w _x _a ' (i.e., 'it is not so that w has been done by x').)
D ₄ NoND ₄ i':	D ₄ Nw _x _a —* ND ₄ w _x _a . (From 'D ₄ Nw _x _a ' (i.e., 'not w has been done for x'), it is assumed to be valid to infer 'ND ₄ w _x _a ' (i.e., 'it is not so that w has been done for x').)

Definitions

- D24d:** $D24wxy = df K-D2wx-D4wy$
 ('w has been done by x for y' is equal to by definition 'w has been done by x and w has been done for y'.)
- D42d:** $D42wxy = df K-D4wx-D2wy$
 ('w has been done for x to y' is equal to by definition 'w has been done for x and w has been done by y'.)
- RighTd:** $RighT-wxy = df OD24wxy$
 ('y has a right that w with respect to x' is equal to by definition 'it is obligatory that w be done by x for y'.)
- DutYd:** $DutY-wxy = df OD42wxy$
 ('y has a duty to w with respect to x' is equal to by definition 'it is obligatory that w be done for x by y'.)
- NorighTd:** $NorighT-wxy = df NOD24wxy$
 ('y has a noright that w with respect to x' is equal to by definition 'it is not obligatory that w be done by x with respect to y'.)
- PrivilegEd:** $PrivilegE-wxy = df NOND42wxy$
 ('y has a privilege to w with respect to x' is equal to by definition 'it is not obligatory that it not be so that w is done for x by y'.)

These definitions of RighT, DutY, NorighT, and PrivilegE lead to the relationships specified by Hohfeld as summarized in Figure 2.

Figure 2

It is obligatory that
w be done by x for y.

(OD24wxy)

RighT-wxy

*
*
,
,
,
,
*
*

It is obligatory that
w be done for y by x.

(OD42wyx)

DutY-wyx

*
*
,
,
,
,
*
*

***-----**

N-NorighT-wxy ***-----** N-PrivilegE-Nwyx

(NNOD24wxy)

It is not the case that
it is not obligatory that
w be done by x for y.

(NNOD42wyx)

It is not the case that
it is not obligatory that
w be done by y for x.

Further exploration into the Hohfeldian system to formalize the concepts of Conditional rightT and PoweR is beyond the scope of this article. Formalization of these two concepts and other associated with them requires introduction of functional calculus, as well as the concept of time. This will be treated in a subsequent article.

CONCLUSION

The first part of Hohfeld's system of analysis—namely the part that deals with RightS, DutieS, NorightS, and PrivilegeS—is formalized in the preceding pages after detailed consideration of the problems involved in defining ObligationN, which in turn is used in defining RighT and the other three Hohfeldian concepts. Six criteria are proposed for testing the adequacy of any definition of ObligationN, and it is shown that the difficulties of most definitions of 'O' are linked with how if-then is formalized in the various logical systems considered. Certainly, one may wish to add to these criteria and further refine the concept of ObligationN, or one may opt for a different outcome with respect to the six properties explored. The important point is not that a complete and final stipulation of ObligationN (and

the other concepts that depend upon it) shall be definitively achieved in this article, but rather that the process of carefully arriving at such definitions be illustrated. To the extent that other efforts are similarly careful, the research endeavors and analyses of legal scholars can become more cumulative. We would do well to profit from the experience of the natural sciences in this respect and ever recall that . . .

a dwarf sitting on the shoulders of a giant
can see farther than the giant.

FOOTNOTES

1. Wesley N. Hohfeld, *Fundamental Legal Conceptions as Applied in Judicial Reasoning*, 23 YALE L. J. 16, 29 (1913).

2. Felix S. Cohen, *Transcendental Nonsense and the Functional Approach*, 35 COLUM. L. REV. 809, 824-829 (1935).

3. John Chipman Gray, *NATURE AND SOURCES OF THE LAW* vii (1909).

4. Examples of 'right' used in the various senses are included in the Hohfeld article. See, e.g., notes 32, 34, 36a, 65, and 96.

5. Hohfeld, 31-32.

6. Hohfeld, 32.

7. Hohfeld, 32.

8. Terminating a legal relation is one of the two kinds of change possible; the other is creating a legal relation. One of the consequences of Hohfeld's stipulations about fundamental legal conceptions is that the creation of one legal relation will always be the termination of another, and vice-versa.

9. Hohfeld, 45.

9a. For one of the best summaries and examples of careful analysis in the more recent literature on the Hohfeldian system, see Alan D. Cullison, *Logical Analysis of Legal Doctrine: The Normative Structure of Positive Law*, 53 IOWA L. REV. 1209-1268 (1968).

9b. What follows has been so heavily influenced by a multitude of discussions with Alan Ross Anderson down through the years that it is difficult to separate out all that borrows and benefits from these discussions. Suffice it to say that the benefit and influence has been extraordinary; what is presented is my own interpretation of these dialogues in light of my own perspectives about the legal decision making process.

10. Alan R. Anderson, *THE FORMAL ANALYSIS OF NORMATIVE SYSTEMS*, 29, note 36, Tech. Rep. No. 2, Contract SAR/Nonr-609, U.S.O.N.R. Group Psychology Branch, New Haven, 1956.

11. Georg von Wright, *Deontic Logic*, 60 MIND 1, 2 (1951).
12. Frederic B. Fitch, SYMBOLIC LOGIC: AN INTRODUCTION 20-63, Ronald Press, New York, 1952.
13. Irving M. Copi, SYMBOLIC LOGIC 264-265, MacMillan, New York, 1967 (3d ed.).
14. Frederic B. Fitch, *Natural Deduction Rules for Obligation*, 3 AMERICAN PHILOSOPHICAL QUARTERLY 1 (1966).
15. Alan R. Anderson and Nuel D. Belnap, Jr., *The Pure Calculus of Entailment*, 27 JOURNAL OF SYMBOLIC LOGIC 19 (1962).
16. Alan R. Anderson, COMPLETENESS THEOREMS FOR THE SYSTEMS E OF ENTAILMENT AND EQ OF ENTAILMENT WITH QUANTIFICATION, Tech. Rep. No. 6, Office of Naval Research Contract No. SAR/Nonr-609(16), New Haven, 1959.
17. Anderson (1959).
18. See Frederic B. Fitch, SYMBOLIC LOGIC: AN INTRODUCTION, pp. 76-77, for a discussion of the sense in which the idea of *laws of nature* is used here and its relationship to the concept of logical necessity.